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NONLINEAR ANALYSIS OF PILE DISPLACEMENT USING THE FINITE ELEMENT METHOD

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A b s t r a c t

The objective of this paper is a nonlinear analysis of the displacement pile using the finite element method. The results of computer simulation are compared with the *in-situ* test of static load. The authors made an attempt to check this process in order to verify applied computer models in terms of consistency of results. In FEM simulation there are used: actual models of constitutive relations; forming an appropriate geometry; initial and boundary conditions and area discretization of finite elements mesh. The paper is of theoretical character with the results of numerical analysis of pile displacement. Numerical calculations carried out by the authors in this study were conducted using ABAQUS package, which is used for solving problems in soil mechanics.

Keywords: Soil mechanics, computer modelling, finite element method, constitutive soil models

S t r e s z c z e n i e

Artykuł poświęcony jest zagadnieniu nieliniowej analizy osiadania pala w gruncie uwarstwionym pod obciążeniem charakterystycznym od budowli. Przedstawiono problemy odpowiedniego doboru modeli konstytutywnych gruntu, posługując się systemem metody elementów skończonych ABAQUS. Parametry warstw gruntu zostały pomierzone *in-situ* przy zastosowaniu sondy statycznej CPTU i na tej podstawie przyjęto stałe materiałowe modeli teorii plastyczności zastosowanych do opisu równań konstytutywnych. Geometrię warstw gruntu wprowadzono na podstawie przekrojów geotechnicznych. Artykuł ma charakter zarówno teoretyczny (przez dyskusję równań konstytutywnych), jak i praktyczny (przez porównanie wyników symulacji z wynikami testu statycznej nośności pala).

Słowa kluczowe: Mechanika gruntów, metoda elementów skończonych, równania konstytutywne gruntu

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1. Introduction

Pile foundations are widely used in engineering practice. In the traditional approach, the designing of pile foundations consists in checking the limit states. It is fundamental to recognize a soil structure and gain knowledge of a real interaction of the pile that transfers the load with the surrounding soil. Eurocode 7 and the PN-83/B-02482 standard recommend to apply test static load to assess bearing capacity of piles and pile foundations. Currently, an alternative way to discover and understand complex processes in the soil environment are computer simulations. However, this requires a precise description of the area geometry and the adoption of advanced models of materials and load. It is important here to compare a numerical solution with the results obtained in in-situ tests.

In the object analysed in the paper the pile foundations made in the CFA technology were used. It is one of the most popular technologies in Poland.

2. Constitutive models

Constitutive soil models are sets of equations representing the relationship between the stress and strain tensor. Constitutive relations should reproduce the complexity of the soil environment, which consists of three-phase soil property, strong nonlinearity, anisotropy, plastic strengthening and weakening, dilatancy, variable compressibility factor and many more. In this paper, the authors present constitutive models available in the ABAQUS software.

2.1. The Mohr-Coulomb model

The Mohr-Coulomb model (MC) represents the linear envelope of the dependence of normal stress σ from shear stress σ, determining the shear strength. The Mohr Coulomb plasticity condition can be written as an equation describing the six planes forming a distinctive pyramid in the principal stresses space.

$$F = \frac{1}{2}(\sigma_{max} - \sigma_{min}) + \frac{1}{2}(\sigma_{max} + \sigma_{min})\sin(\phi) - c \cdot \cos(\phi) \tag{2.1}$$

where:

- σ_{max} – maximum compressive stresses,
- σ_{min} – minimum compressive stresses.

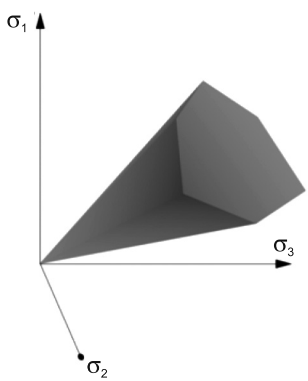

Fig. 1. The MC criterion in the space of stress principal components

2.2. The Drucker–Prager model

The Drucker–Prager model (DP) surface is a cone in the space of the principal stresses, it has a smooth shape on the octahedral plane.

The linear equation of plasticity function:

$$F = t - p \cdot \mathrm{tg}(\beta) - d = 0 \tag{2.2}$$

where:

β – an angle indirectly related to the angle of internal friction – it can be determined experimentally or by the equation relating it to φ:

$$\mathrm{tg}(\beta) = \frac{3 \cdot \sqrt{3} \cdot \mathrm{tg}(\phi)}{\sqrt{9 + 12 \cdot \mathrm{tg}^2(\phi)}} \tag{2.3}$$

d – parameter includes the cohesion c and internal friction φ given by

$$d = \frac{3 \cdot \sqrt{3} \cdot c}{\sqrt{9 + 12 \cdot \mathrm{tg}^2(\varphi)}} \tag{2.4}$$

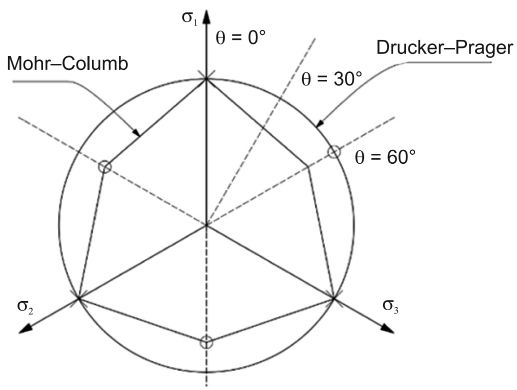

Fig. 2. The Lode angle Θ on the deviatoric plane (p = const) and directions of triaxial compression and tension

3. Soil modelling – FEM formulation

Soil is a porous material with a complex structure – a skeleton (for mineral grains or organic particles) and a space filled with air and/or water (or other liquid). Soils are modelled as a single-, two- or three-phase materials depending on the volume ratio of the skeleton, air or water in the pores. The single phase model maps the soil as a mixture with mechanical and physical parameters at each point of the area. The two-phase model is used for solving problems related to fully saturated soils or the case of dry soils. To analyse the issues presented in this paper, the authors took into account the two-phase property of the soil environment.

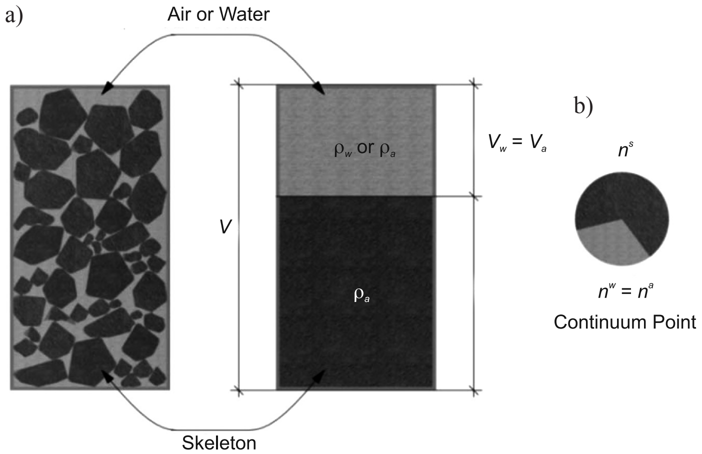

Fig. 3a) The two-phase model of soil, $n^s + n^w = 1$ (air or water and skeleton), b) Continuum point

Using the finite element simulation of displacement pile requires application of appropriate modelling of pile elements interactions with the surrounding soil. In computer simulation one can use: the case without a contact (the soil elements and adjacent pile elements are forced to move together) or the case with a contact. The main advantages of using the contact are: 1) frictional behaviour of a pile with soil is fully represented in the model; 2) different movements of the pile elements and soil (slip) are possible. It should be noted that pile elements are relatively rigid to the surrounding soil deformations. The surface of pile elements that are in contact with the soil elements is referred to as the main surface (*master*). The surface of the soil elements in contact is set as the secondary surface (*slave*). In the ABAQUS program these surfaces are called *contact pairs*. The contact pair representing the interaction of a pile with the soil is shown in Fig. 3.

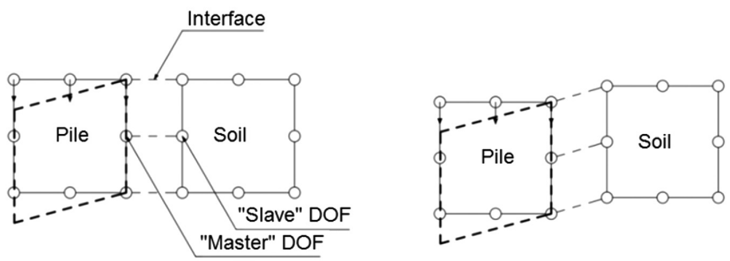

Fig. 4. The contact pair scheme in ABAQUS

The contact behaviour of the two surfaces is determined by the Coulomb law of sliding friction. The Coulomb model of friction applies to the maximum allowable friction (shear), including the normal stress at the interface between the two surfaces.

The force of sliding friction between two bodies is proportional to the normal component of the force, keeping bodies in contact as given by the formula:

$$\tau_{crit} = \mu\, p \tag{3.1}$$

where:

τ_{crit} – the critical shear stress on the interface,
μ – the friction coefficient,
p – normal component of the stress between the two surfaces.

If the frictional stress balances the tangential stress, it is a condition called "gluing".

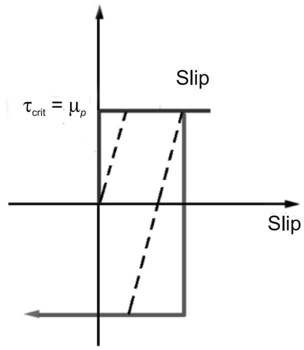

Fig. 5. The Coulomb model of friction

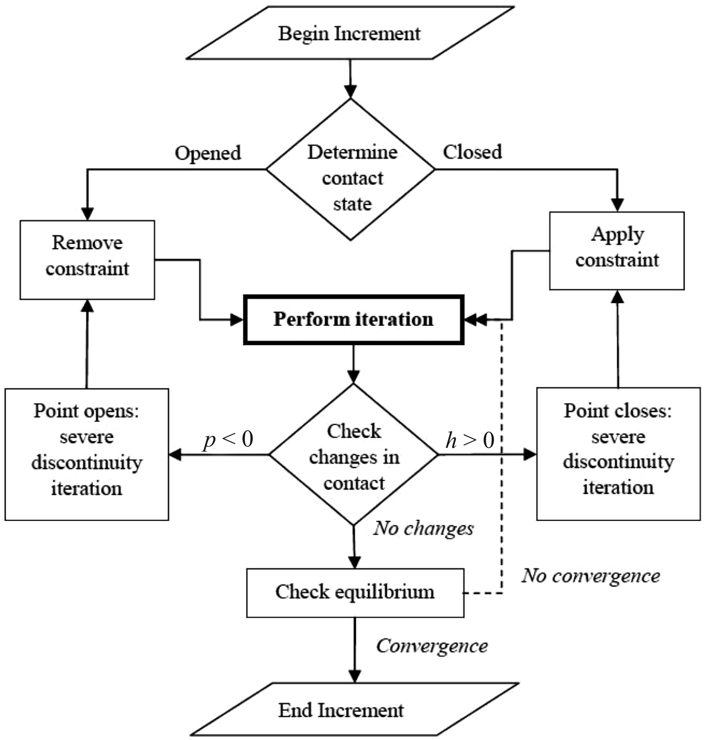

Fig. 6. The master-slave contact algorithm in ABAQUS

The contact algorithm implemented in the ABAQUS program can be summarized as follows:

1. Determination of the condition of all contact interactions by checking whether each slave node (slave) is “open” (the distance from the master node is greater than zero) or “close” (the distance is equal to zero) at the beginning of each increment.
2. The imposition of a restriction for each closed slave node, while it is still closed or removing the restriction, if the contact status of the node changes from closed to opened; determination whether a closed node is glued or slipping.
3. Contact calculating conditions are provided by iteration in each slave node up to limits control. Having updated the condition of contacts, a check of forces and/or moments balance for this increase is made.
4. Checking the pressure *p* and distance *h* in each slave node. If the pressure is negative after the iteration, the status of the contact in this slave node is changed from “closed” to “opened”. On the other hand, if the pressure is negative and the distance from the master node after the iteration is zero, the contact of each node is changed from “opened” to “closed”. In both cases, the condition is specified as a “severe discontinuity iteration” and the balance is not checked.
5. Updating a contact reduction after the first iteration – repeating the iteration procedure until there are changes in the contact condition.

4. The geometry model

Modelling of soil environment in terms of FEM includes mapping the actual state by adopting selected constitutive relations, creation of the appropriate geometry, boundary conditions. The theoretical soil area is unlimited half-space. The authors decided to apply the standard approach – the model with finite dimensions.

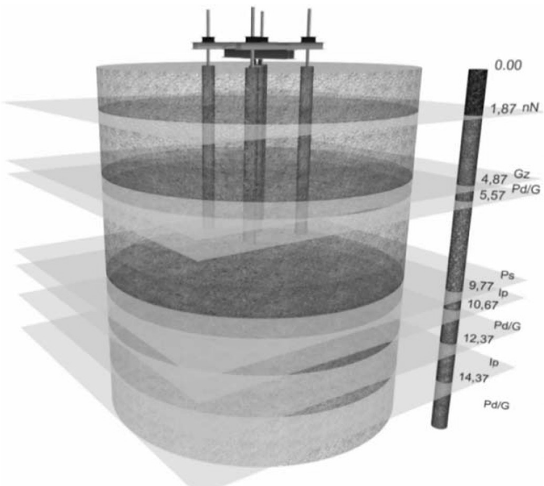

Fig. 7. The physical model of a pile

Due to the location of anchor piles at a distance of 2.75 m from the axis of the loaded pile, the authors ignored their impact in the numerical model. ABAQUS computing system was applied as an axisymmetric model of a cylindrical pile with a diameter of 0.75 m and a length of 8 m. The radius of the soil area was adopted as 8 m (length of the pile) and the model height was 16.83 m (two times greater than the length of the pile) on the basis of a test borehole. Symmetry conditions allowed modelling of a half section of the pile and surrounding soil.

The finite element grid was formed with 1025 elements. The pile was modelled using axisymmetric CAX4R type elements (4-*nodal bilinear axisymmetric quadrilateral*) while the soil by CAX4P type elements (4-*nodal axisymmetric quadrilateral, bilinear displacement*, *bilinear pore pressure*). These elements are available in the library of ABAQUS/Standard elements. The contact between the pile and the soil was modelled using the *master-slave* contact. The coefficient of friction $\mu = \mathrm{tg}\phi = 0.39$ was adopted for an average value of the angle ϕ of soil layers.

5. The material model

The elastic coefficients were chosen on the basis of the PN-81/B03020 standard and geotechnical tests.

Table 1

Elastic parameters of soils

Soil	Oedometric modulus of elasticity, M [MPa]	Modulus of elasticity, E [MPa]	Poisson's ratio v
Ne (non-building fill)	33.5	25.1	0.3
Cc (compact clay)	34	23.7	0.32
Fs/Cl (fine sand/clay)	60	44.5	0.3
Ms (medium sand)	130	104	0.27
Sl (sand loam)	18	10.2	0.37
Fs/Cl (fine sand/clay)	60	44.5	0.3
Sl (sand loam)	18	10.2	0.37
Fs/Cl (fine sand/clay)	60	44.5	0.3

The values of oedometric modulus of elasticity were chosen from PN-81/B03020 standard on the basis of geotechnical tests. Due to the nature of the applied load, the authors decided to use the initial value. The Poisson's ratio has been chosen on the basis of Table 3 of the PN-81/B03020 standard. The modulus of elasticity was calculated using the formula:

$$E = M \frac{(1+\nu)\cdot(1-2\nu)}{(1-\nu)} \tag{5.1}$$

Volume weights of individual soil layers have been adopted on the basis of the PN-81/B03020.

Using the Mohr–Coulomb model, the authors selected the coefficients listed in Table 2.

Table 2

Plastic coefficients of the Mohr-Coulomb model

Soil	ϕ [°]	c [kPa]	ψ [°]
Ne (non-building fill)	15	0	0
Cc (compact clay)	10	11.3	0
Fs/Cl (fine sand/clay)	30.3	0	0.3
Ms (medium sand)	34.2	0	4.2
Sl (sand loam)	12.4	50	0
Fs/Cl (fine sand/clay)	30.3	0	0.3
Sl (sand loam)	12.4	50	0
Fs/Cl (fine sand/clay)	30.3	0	0.3

The angle of internal friction and cohesion were selected on the basis of the PN-81/B03020. The angle of dilatancy has been chosen according to literature. For the application of the Drucker–Prager model, the coefficients given in Table 3 were used.

Table 3

Plastic coefficients of the Drucker–Prager model

Soil	β [°]	d [kPa]	ψ [°]
Ne (non-building embankment)	24	0	0
Cc (compact clay)	17	19	0
Fs/C (fine sand/clay)	40	0	0.3
Ms (medium sand)	43	0	4.2
Sl (sand loam)	20	84	0
Fs/C (fine sand/clay)	40	0	0.3
Sl (sand loam)	20	84	0
Fs/C (fine sand/clay)	40	0	0.3

6. The load model

The analysis was divided into five steps. The GEOSTATIC step, CONSOLIDATION steps, mapping the process of loading and unloading of the pile during the test. In the GEOSTATIC step, for all elements in the model, the dead load was applied by the gravitational acceleration equal to $g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2}$ using an automatic scheme of time increment under small deformation conditions. The first time step (initial time) was: $t_i = 1 \cdot 10^{-1}$ with the total duration of $t = 1$. Automatic increment was used in the analysis for time steps from the range: $\Delta t_{min} = 1 \cdot 10^{-5}$and $\Delta t_{max} = 1$. Water pressure changes in the pores at any increment are set using the UTOL option. If the maximum change in the pore water pressure at any node is greater than the predetermined tolerance, the increment is repeated with the proportional reduction of the time step. Otherwise, the time step increases. The total time of steps (2–5) SOILS, CONSOLIDATION is $t = 770$ min according to the actual time of sample testing of the pile. The pile was loaded with the concentrated force of a value1085 kN applied to the pile head, scaled in the following steps of the analysis according to the diagram.

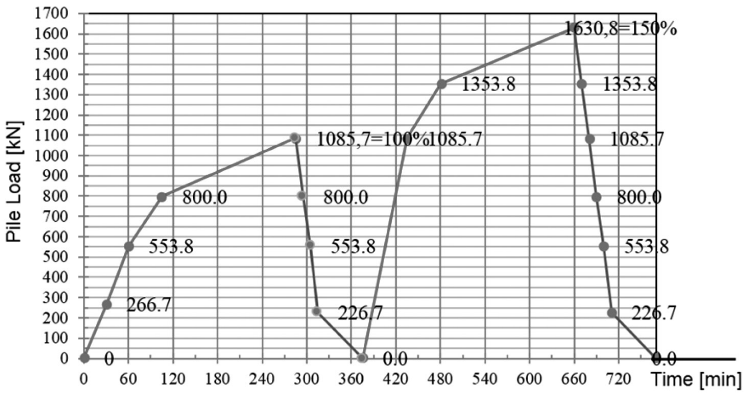

Fig. 8. The load range in the following steps of the analysis

In step 2 the pile is loaded to 100% of its bearing capacity of ULS. In step 3 the pile is unloaded from 100% to 0% of a value at the time $t = 90$ min. The total duration of step 3 is $t = 375$ min.

In the next step, the pile is re-loaded to 150% at the time $t = 285$ min The total duration after step 3 is $t = 660$ min

In the last step, the force value drops to 0% at the time $t = 770$ min.

7. Results of the analysis

The results of the numerical analysis of two constitutive models (MC and DP) are presented with the field tests results.

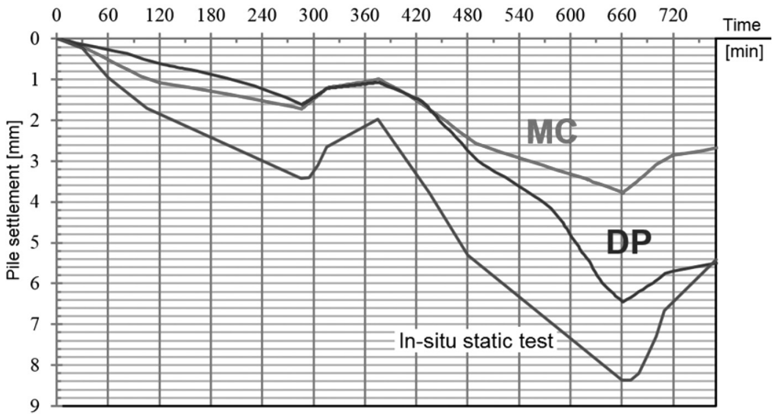

Fig. 9. Comparison of displacements

Fig. 9. presents the results of the two computer models and the in-situ static test results. It should be noted that the shape of the graph for the MC model is similar to the shape of actual displacements. The differences result from:

– the application of constitutive models,
– incorrect selection of soil parameters resulting from errors and inaccuracies in geological studies – these parameters should be calibrated using e.g. an inverse method,
– approximate description of geological strata due to a certain distance of the test pile from a geological surveys borehole,
– adopted average value $\mu = \mathrm{tg}\phi$ over the length of the pile.

The plot shown in Fig. 10. representing the comparison of the results for the DP model shows displacements very similar to the real values – in step 4 (150%) for the DP model the displacements reach a maximum value of 6.45 mm compared to 8.37 mm found in the in-situ tests. After the last step of the analysis, the permanent displacements are equal to 5.49 mm for DP, while they are 5.41 mm in the in-situ tests. However, the shape of the graph shows different behaviour in comparison to the actual tests.

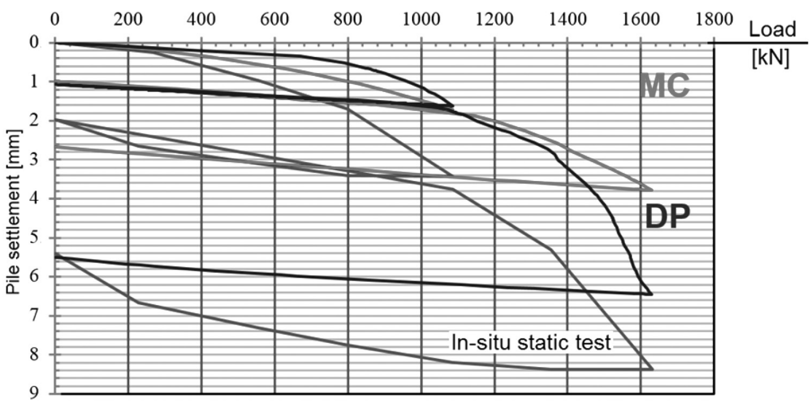

Fig. 10. The comparison of graphs showing the displacement-load dependence

Analysing Fig. 10, it should be noted that in the first phase of the in-situ load application – the 2nd step of the analysis – their shapes are similar. In step 4, the applied force increases from 0 to 150% NP within the same time interval as in step 2. At this stage of the analysis, the MC and DP soil response models are divergent.

In the DP model the displacements are greater, resulting in larger permanent (plastic) deformation that can be noticed after completing the analysis. The authors concluded that the Drucker-Prager constitutive equation applied as a model of soil gives a solution more similar to the static load field tests.

References

[1] *Analysis of Geotechnical Problems with Abaqus*, ABAQUS, Inc., 2006.
[2] *ABAQUS User's Manual*, 2011.
[3] Cichoń C., *Computer Methods in the Linear Structural Mechanics*, Wydawnictwo Politechniki Krakowskiej, Kraków 2002.
[4] Gryczmański M., *Load Path Methods in the Analyses of Soil Mechanics Problems*, 2002.
[5] Helwany S., *Applied Soil Mechanics: with ABAQUS Applications*, John Wiley & Sons, Inc., 2007.
[6] Gwizdała K., *Piles and Pile Foundations Settlement. Seminar – Pile Foundations Issues*, Gdańsk 2004.
[7] Han L.H., *A Modified Drucker-Prager Cap Model for the Compaction Simulation of Pharmaceutical Powders*, International Journal of Solids and Structure, 45 (10), 2008, 3088-3106
[8] Cudny M., Binder K., *Criteria of Soil Shear Strength in Geotechnology*, Inżynieria Morska i Geotechnika 6, 2005, 456-465.
[9] Ivorra S., *Drucker–Prager yield criterion application to study the behaviour, of CFRP confined concrete under compression*, XXXVII IAHSWorld Congress on Housing, October 26–29, Santander, Spain 2010.
[10] Schümann B., *Modelling of soils as multiphase-materials with Abaqus*, SIMULIA Customer Conference, 2010.
[11] Torben Pichler T.P., *High-Performance Abaqus Simulations in Soil Mechanics*, SIMULIA Community Conference, 2011.
[12] Wiłun Z., *Introduction to Geotechnology*, WKŁ, 2005.
[13] Wrana B., *Soil Dynamics. Computational Models*, Wydawnictwo Politechniki Krakowskiej, Kraków 2012.
[14] Zienkiewicz O.C., Taylor R.T., *The Finite Element Method*, Butterworth-Heineman, 2000.