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COMPARISON OF SLOPE STABILITY PREDICTIONS BY GRAVITY INCREASE AND SHEAR STRENGTH REDUCTION METHODS

PORÓWNANIE PROGNOZ STATECZNOŚCI SKARPY METODAMI ROSNĄCEJ GRAWITACJI I REDUKCJI WYTRZYMAŁOŚCI

A b s t r a c t

The finite element method has become the most widespread method of analyzing slope stability. Its advantage over the limit equilibrium method (the method of slices) has been proven in many publications. The most commonly applied FEM computational scheme in slope stability calculations is the shear strength reduction (SSR) method. Another way of analysing slope stability by FEM is the gravity increase (GI) method. The latter method may not be applied unconditionally. The paper contains a comparison of results obtained by the SSR, GI and modified Bishop's methods. A good agreement of the results produced by the SSR and the modified Bishop's methods has been confirmed. Overestimation of the safety factor by the GI method in the case of using the linear Mohr-Coulomb criterion has been shown.

Keywords: finite element method, limit equilibrium method, slope stability

S t r e s z c z e n i e

Metoda elementów skończonych stała się powszechnie stosowanym narzędziem do analizy stateczności skarpy. Jej przewagę nad metodami równowagi granicznej (metodami paskowymi) wykazano w licznych publikacjach. W ramach analiz stateczności MES najczęściej stosowana jest strategia redukcji wytrzymałości na ścinanie. Innym podejściem jest strategia rosnącej grawitacji, jednak nie może ona być stosowana bezkrytycznie. W artykule zaprezentowano porównanie wyników obliczeń wartości współczynnika bezpieczeństwa metodą redukcji na ścinanie, rosnącej grawitacji i uproszczoną metodą Bishopa. Pokazana została zbieżność wyników uzyskiwanych metodą redukcji wytrzymałości na ścinanie oraz metodą paskową Bishopa oraz przeszacowanie stateczności skarpy w metodzie rosnącej grawitacji w przypadku zastosowania tradycyjnego liniowego warunku Coulomba-Mohra.

Słowa kluczowe: metoda elementów skończonych, metoda równowagi granicznej, stateczność skarpy

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Symbols

F_s	–	safety factor
$\mathbf{d}$	–	displacement vector
$\mathbf{B}(\mathbf{x})$	–	strain-displacement matrix at a spatial point $\mathbf{x}$
$\mathbf{N}(\mathbf{x})$	–	matrix of shape functions at a spatial point $\mathbf{x}$
ρ	–	density
$\mathbf{g}$	–	gravity
$\dot{\mathbf{g}}$	–	gravity rate
t	–	parametric time
$\mathbf{t}$	–	surface tractions prescribed on the part S_t
$\boldsymbol{\sigma}$	–	stress corresponding to the displacement $\mathbf{d}$
c	–	cohesion
ϕ	–	friction angle

1. Introduction

Slope stability can be defined as a condition in which the body forces generated by gravity together with the load at the top (crest) and internal forces in the massif are in equilibrium. Instability occurs if this condition is not met. Instability may arise as a result of the mechanism developed along a continuous zone, often called a slip surface.

Various approaches to the solution of the slope stability problem have been classified into four groups [5]:
1) the limit equilibrium method;
2) the slip line method;
3) the finite element method;
4) a combination of the above 3 groups.

A traditionally slope stability problem was analysed by means of the limit equilibrium method (method of slices by Fellenius, Bishop, Janbu, Morgenstern and Price, Spencer). Description of these methods can easily be found in the literature (e.g. [3–5]). Limit equilibrium methods are still under development. The closed-form solutions satisfying both equilibrium of moments and forces in 2D [11] and 3D [12] were proposed recently.

To assess the level of safety of a slope (natural or cut), shear stresses acting in the soil body are compared with the shear strength (ultimate shear stress) on a priori assumed surface and the parameter called the factor of safety is calculated:

$$F_s = \frac{\text{shear strength of soil}}{\text{shear stress required for equilibrium}} \tag{1}$$

The limit equilibrium methods mentioned above suffer from substantial drawbacks:
- they do not take into account the history of slope formation, i.e. stress-strain (loading) history;
- they assume a given shear resistance along a priori assumed slip line (most often circular);
- the methods provide no information as to the magnitudes of strains within the slope nor any indication about how they may vary along the slip surface;

- as a consequence, there is no guarantee that the shear resistance will take the peak or residual value simultaneously on the whole slip surface which is the assumption of all limit equilibrium methods;
- a rigid-plastic constitutive model with the Mohr-Coulomb limit state surface is commonly used in analyses;
- as a consequence, failure occurs only if the stress path reaches the limit state surface whereas in some real cases, it may occur before reaching the failure condition.

Numerical methods for solving boundary value problems offer much more powerful tools for analysing slope stability [1, 6]. With the finite element method slope, stability may be analysed by making use of constitutive relations and the procedure called the shear strength reduction. Determination of the factor of safety consists in successive decreasing of strength parameters with respect to their initial values. A failure criterion met along an arbitrary continuous line indicates the loss of stability which is close to reality.

Moreover, as noted in [15], the limit equilibrium method is of an approximate and arbitrary nature and the results obtained from this method are neither upper bounds nor lower bounds on the true collapse loads. The finite element method can be used to calculate upper and lower bound solutions for slope stability, though some special approach is necessary [15]. This attempt delivers evaluations for the collapse load calculated in limit analysis.

In the paper, two alternative methods of finite element stability analysis are presented. In the first, the strength characteristics of the soil mass are held constant, and the gravitational loading on the slope system is increased until failure is initiated by a well-defined mechanism. In the second approach, the gravity loading on the slope system is held constant, while the strength parameters of the soils are gradually decreased until a well-defined failure mechanism develops. The first method is called the gravity increase method [8, 14] while the second is the well known shear strength reduction method [1, 3, 6, 8–10, 14, 15]. The results from these two approaches are compared with the results of the modified Bishop's limit equilibrium method.

2. Force equilibrium in the non-linear finite element method

The response of soil slope subjected to gravitational loading is treated as a general materially non-linear elliptic boundary value problem. The load is applied in incremental steps. In the nth step, the equilibrium between internal and external forces within the soil body is searched, which due to the non-linearity of the soil model, means that the residual forces meet the requirement:

$$\mathbf{r}_n = \mathbf{f}_{\text{int}}\left(\mathbf{d}^{(n)}\right) - \mathbf{f}_{\text{ext}}\left(\mathbf{d}^{(n)}\right) = 0 \tag{2}$$

where:

$$\mathbf{f}_{\text{int}}\left(\mathbf{d}^{(n)}\right) = \int_V \mathbf{B}^T(\mathbf{x})\boldsymbol{\sigma}\left(\mathbf{B}(\mathbf{x})\mathbf{d}^{(n)}\right)dV \tag{3}$$

$$\mathbf{f}_{\text{ext}}\left(\mathbf{d}^{(n)}\right) = \int_{S_t} \mathbf{N}^T(\mathbf{x})\mathbf{t}^{(n)}dS + \int_V \rho\mathbf{N}^T(\mathbf{x})\mathbf{g}^{(n)}dV \tag{4}$$

$\mathbf{d}^{(n)}$ – displacement vector at n^{th} step,
$\mathbf{B}(\mathbf{x})$ – strain-displacement matrix at a spatial point $\mathbf{x}$,
$\mathbf{N}(\mathbf{x})$ – matrix of shape functions at a spatial point $\mathbf{x}$,
ρ – density,
$\mathbf{g}$ – gravity,
$\mathbf{t}$ – surface tractions prescribed on the part S_t,
$\boldsymbol{\sigma}$ – stress corresponding to the displacement $\mathbf{d}^{(n)}$.

The slope is stable with respect to the applied loads as long as equilibrium can be achieved between the internal and external forces, than a solution to equation (2) exists. When this balance can no longer be achieved due to increased gravity loading and achieving soil strength, the slope becomes unstable, since the equilibrium solution satisfying equation (2) no longer exists.

3. Gravity increase (GI) method

The analysis of stability by the gravity increase method assumes the external forces increase due to increasing gravity **g** and the equilibrium solution satisfying equation (2) can no longer be obtained. Monotonically increasing gravity brings external forces on the edge of stability when the strength of the soil is reached.

Gravity increases according to the formula:

$$\mathbf{g} = \dot{\mathbf{g}} \cdot t \tag{5}$$

where:

$\dot{\mathbf{g}}$ – a prescribed vector specifying the direction of gravity loading and its rate of increase with time, and t is a parametric time variable.

Prescribed in this manner, gravitational acceleration vector $g(t)$ increases and the limit analysis problem reduces simply to finding the largest time $t = t_{\text{limit}}$ for which a global equilibrium solution of equation (2) exists. The limiting acceleration due to gravity in the system is then:

$$\mathbf{g}_{\text{limit}} = \dot{\mathbf{g}} \cdot t_{\text{limit}} \tag{6}$$

The time t_{limit} is not a known a priori. It is approached asymptotically. For values of $t > t_{\text{limit}}$ solution of equation (2) does not exist.

Since gravitational loading induces slope failure, the gravity-based factor of safety against slope failure is given by:

$$F_{s\,gi} = \frac{g_{\text{limit}}}{g_{\text{actual}}} \tag{7}$$

where:

g_{actual} – representative actual acceleration due to gravity in the slope analysed, i.e. 9.81 m/s^2.

The value of the safety factor is greater than unity for a stable slope. The higher the value of safety factor, the more stable the slope is.

It has been found in [8] that a good measure of slope safety is to associate g_{limit} with the abrupt increase of acoustic emission rate or a dramatic increase in the nodal displacement within the elements. Nevertheless, in this paper nonconvergence of the FEM solution and displacement variation in the slope body is adopted.

4. Shear strength reduction (SSR) method

In this technique, the basic continuum equilibrium problem and the corresponding finite element formulation for each stage of analysis is the same as for the gravity increase method. A series of trial factors of safety are used to adjust the cohesion, c, and the friction angle, ϕ, of soil as follows:

$$c_{\text{trial}} = \frac{1}{F} c \tag{8a}$$

$$\varphi_{\text{trial}} = \arctan\left(\frac{1}{F} \tan \varphi\right) \tag{8b}$$

The adjusted cohesion and friction angle of the soil layers are re-inputted in the model for equilibrium analysis. The factor of safety is sought when the specific adjusted cohesion and friction angle make the slope unstable.

In general, the shear strength parameters of the soil can be reduced by utilizing a monotonically decreasing time function to govern the shear strength properties of the soil mass. A typical shear strength parameter X for the soil mass is governed in time as follows:

$$X(t) = X_{\text{base}} \cdot f(t) \tag{9}$$

where:

X_{base} – the actual strength parameter and t is again a parametric time variable (pseudo-time).

In such a problem, gravity loading is applied to the soil mass and remains unchanged after the stability analysis is launched. The initially high values of soil parameters determining its strength decrease monotonically until solution of equation (2) can no longer be found. Thus, the problem again concerns finding the maximum value of parametric time $t = t_{\text{limit}}$ for which the boundary value problem is on the verge of failure. In fact, the shear strength reduction method is a repeatedly solved problem of statics with varying parameters of soils in a model.

The sought time corresponds to the safety factor for the strength reduction method:

$$F_{ssr} = \frac{X_{\text{base}}}{X\left(t_{\text{limit}}\right)} = \frac{1}{f\left(t_{\text{limit}}\right)} \tag{10}$$

5. Numerical analyses

5.1. Assumptions

The stability analyses have been performed for a slope with the crest loaded with 90 kPa placed 5 m away from the edge of the slope. The slope is made of homogeneous clay. The geometry of the slope comprises four heights $H = 6, 12, 20, 40$ m and two angles of inclination $\alpha = 30°$ and $45°$ (Fig. 1).

There were two sets of material parameters assumed for the analyses with the Mohr-Coulomb elastic – perfectly plastic model. Values of elastic modulus $E = 30$ MPa, Poisson's ratio $\nu = 0.3$, density $\rho = 2000$ kg/m^3 are common for both sets. Strength parameters differ for inclinations 30° and 45°. For the 30° slope, cohesion is 10 kPa and the friction angle is 25° whereas for the 45° slope, cohesion is 30 kPa and the friction angle 30°. In both cases, non-associated flow rule has been assumed with a dilatancy angle $\psi = 0°$.

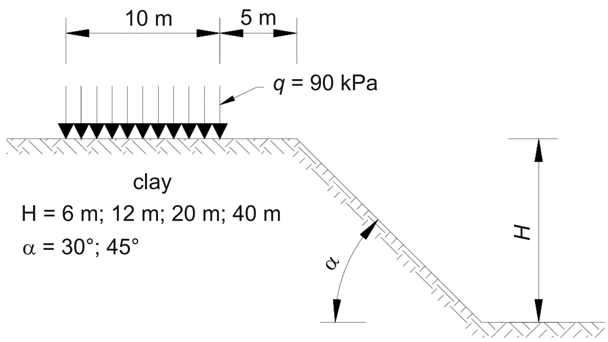

Fig. 1. Shape of slope under analysis

The analyses have been performed with the FEM program Z_Soil [16]. A family of 2D isoparametric elements with 1st order interpolation function is implemented in the program.

To simulate incompressible and highly dilatant plastic media, the Enhanced Assumed Strain method (EAS) has been used in the program.

5.2. Results

Analyses of slope stability have been performed by both shear strength reduction and gravity increase methods. The results have been compared with the results of the limit equilibrium analysis by the modified Bishop's method.

The results of computations by the shear strength reduction method are presented in Fig. 2. Concentrations of shear strain (the second invariant of strain deviator) forming shear bands can be seen. These are incremental values that occur in the last stage of each analysis. At this stage, further reductions of the soil strength give rise to the failure mechanism and divergence in computations.

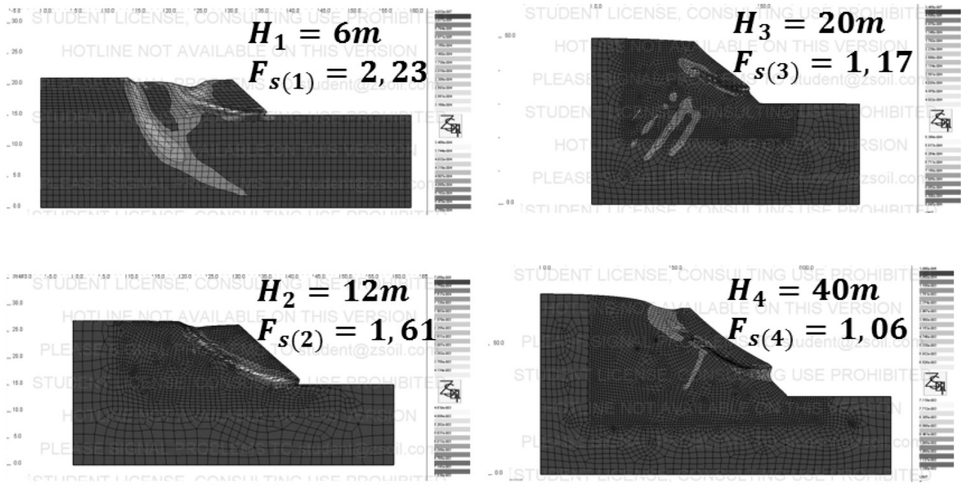

Fig. 2. Deformed models of slope inclined at 45° showing failure mechanisms in the shear strength reduction method

In all cases, the failure mechanisms are well defined. For the 6 m slope, two simultaneous mechanisms are predicted; one starting at the toe, the other deep in the base. The higher the slope is, the higher the failure zone starts on its surface.

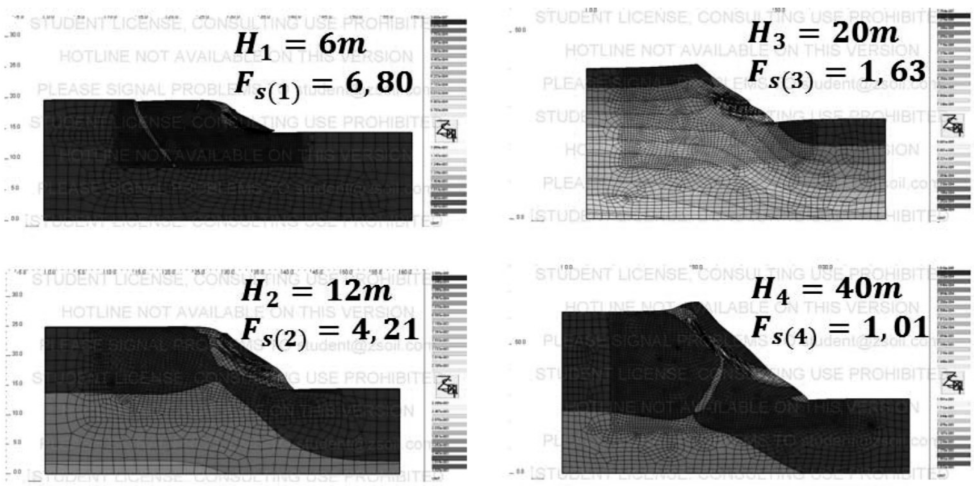

Fig. 3. Deformed models of slope inclined at 45° showing failure mechanisms in the gravity increase method

Results of computations by the gravity increase method for the 45° slope are presented in Fig. 3. Values of the factor of safety are significantly larger than in the shear strength reduction method for slopes up to 20 m. In the case of the 40 m slope, both methods predict virtually identical factors of safety.

As in the case of the shear strength reduction method, two failure zones occur in the 6 m slope. In the 20 m slope, the failure mechanism predicted by the increase gravity method is not unique. There are two failure zones starting at the toe and at one-third of the slope's height.

The results of computations by Bishop's method are close to the results achieved by the shear strength reduction method. For all slope's heights, Bishop's method yields somewhat higher results than shear strength reduction method, but it can be admitted that for homogeneous slopes, the values of the factor of safety predicted by both methods are in good agreement. These findings are comparable to the results reported in, for example [6, 8, 14].

The gravity increase method yields significantly larger factors of safety than the shear strength reduction method. Such results were expected since the traditional Mohr-Coulomb failure criterion has been used in the analysis. This criterion is a linear function of stresses. In such a case, in the absence of additional loads, with increasing gravity, the mean stresses in the soil increase faster than the shear stresses. The rate of strength gain in soil under increasing gravity loading, exceeds the rate of shear stress increase, and the slope does not develop a failure mechanism.

Such a case is depicted in Fig. 4, where a slope of sand inclined at 33 has been analysed until an increase of gravity upto 20 g. The friction angle for sand has been assumed at 35°. It is well known that such a slope is at the verge of stability. The shear strength reduction method yields the factor of safety $F_s = 1.09$ whereas the increase gravity method does not reveal failure mechanism even for a gravity field of 20 g ($F_s = 20$).

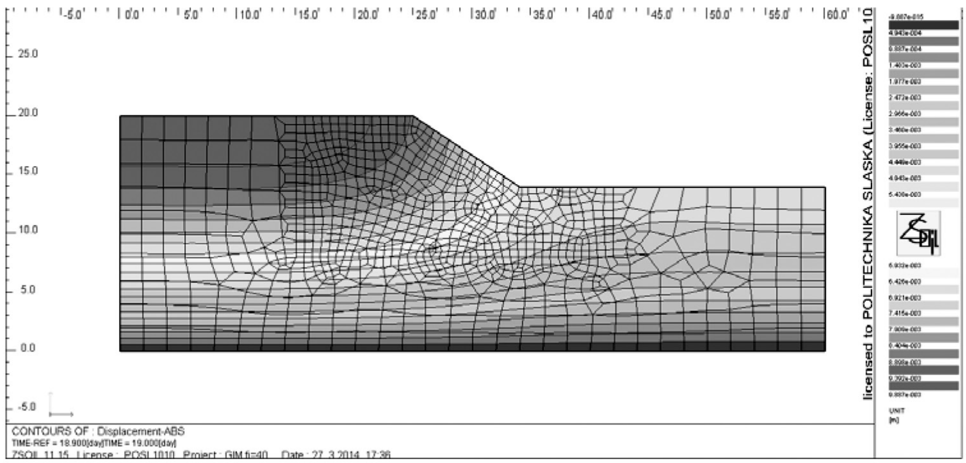

Fig. 4. Displacements of sandy slope inclined at 33° without failure mechanisms in gravity increase method at gravity field increased 20 times

In the problem considered, an additional load of 90 kPa on the crest of the slope is present. In addition, the slope is not comprised of purely frictional material. The cohesive part of the strength is not influenced by the mean stress. Since shear stresses due to gravity and external loading increase faster than the strength in the superficial zone, slip zones develop near the surface of the slope for all heights.

Factors of safety computed with all methods for both repose angles of 30° and 45° are presented in Table 1.

Table 1

Comparison of factors of safety computed by different methods

Height of a slope H [m]	Factor of safety F_s					
	repose angle 45°			repose angle 30°		
	Bishop's method	Shear Strength Reduction	Gravity Increase	Bishop's method	Shear Strength Reduction	Gravity Increase
6	2.33	2.23	6.80	1.67	1.47	10.61
12	1.69	1.61	4.21	1.41	1.30	3.36
20	1.39	1.17	1.63	1.27	1.11	2.01
40	1.11	1.06	1.01	1.09	1.01	1.01

For small height of slope, the strength reduction method produces stability factors smaller (and thus more conservative) than the gravity-loading method of analysis. At larger slope height, both methods tend to give stability factors and failure mechanisms that are somewhat closer to each other.

6. Summary

Overcoming a set of drawbacks that feature the limit equilibrium method, the finite element method came into widespread use within engineering practice. The shear strength reduction method is currently the most frequently used method of analyzing slope stability by the finite element method. Nevertheless, the gravity increase method is also applied in stability analysis.

The criterion that is most often used for judging the slope failure is nonconvergence of the FEM solution, but there are other possible criteria including the formation of a critical failure surface, displacement variation in the slope body, acoustic emission event rate or negative second order work in a closed area within a slope [2, 7, 13]. In the paper, the first of them has been used for both shear strength reduction and gravity increase methods.

From the results obtained from the analyses, it follows that the application of the traditional Mohr-Coulomb failure criterion in the gravity increase method leads to a significant overestimation of a factor of safety in comparison with the shear strength reduction method in the case of relatively low slopes. The discrepancy between the values of the factor of safety for higher slopes (over 20 m) obtained from both methods become smaller.

Once again, the good agreement between factors of safety computed by the shear strength reduction method and the modified Bishop's method has been shown. Such results are well known from the literature [1, 10, 15]. However, it must be remembered that this is valid only for homogeneous slopes.

The present study uses only the simple elastic – perfectly plastic Mohr-Coulomb model. Since many of soil the deposits exhibit softening behaviour, other more advanced models which also take into account this aspect should be considered in stability calculations. Such an analysis should be of special interest in the case of rock slopes. Some efforts have already been put into using a simple brittle damage model in gravity increase FEM slope stability analysis [8].

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