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HISTORY OF DESCRIPTIVE GEOMETRY WITH AN EMPHASIS TO THE BOOM OF DESCRIPTIVE GEOMETRY IN AUSTRO-HUNGARIAN EMPIRE IN THE 19TH CENTURY

HISTORIA GEOMETRII WYKREŚLNEJ ZE SZCZEGÓLNYM UWZGLĘDNIENIEM JEJ ROZKWITU W MONARCHII AUSTRO-WĘGIERSKIEJ

A b s t r a c t

The article shows the development of projection in the Antiquity, the origin of perspective in Renaissance and the development of orthogonal projection from the 16th up to the 18th century before descriptive geometry as a separate discipline of studies was established by Gaspard Monge. Furthermore, the paper presents the expansion of descriptive geometry through Europe in the 19th century with the emphasis on its bloom in the second half of the 19th century in Cisleithania.

Keywords: *descriptive geometry*, *orthogonal projection*, *perspective*, *Cavalier projection*, *stereotomy*, *Gaspard Monge*, *polytechnics in Cisleithania*

S t r e s z c z e n i e

W artykule przedstawione zostały techniki rzutowania w starożytności, początki perspektywy w czasach renesansu oraz rozwój technik rzutowania prostokątnego od XVI do XVIII w., zanim Gaspard Monge uczynił geometrię wykreślną jako osobną naukę. Ponadto w artykule przedstawiono rozprzestrzenianie się geometrii wykreślnej w Europie w XIX w., ze szczególną uwagą na jej rozkwit w drugiej połowie XIX w. w Przedlitawii.

Słowa kluczowe: *geometria wykreślna*, *rzutowanie prostokątne*, *perspektywa*, *rzut kawaleryjski*, *stereometria*, *Gaspard Monge*, *politechniki w Przedlitwaii*

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1. Introduction

Descriptive geometry as a science was formed in the end of the 18th century in France by Gaspard Monge, but various methods of projection were used long before. The development of the projection methods was interesting and very important for the formation of descriptive geometry, which is needed for the work of project architects, builders but also for some doctors and other professions.

2. Use of projection in the Antiquity

2.1. Orthogonal projection

We have only a little extant evidence of the use of projection in the ancient times. Orthogonal projection was used mainly in architecture. The top orthogonal views of buildings or temples were carved into stones just like the front orthogonal views of sculptures or columns. In Fig. 1 we can see a papyrus from about 330–390 BC with two orthogonal projections of an Egyptian sphinx. In Fig. 2 there is a column from the Philae island[1]. This column was carved into a stone about 150 BC at full proportion [4]. In these figures there are always two views, but only one view was often used in this period.

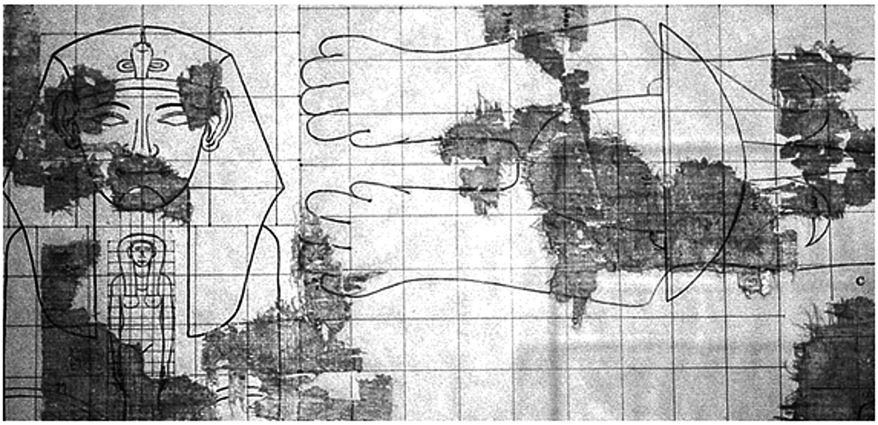

Fig. 1. Papyrus with orthogonal projections of a sphinx [4]

The oldest extant known written comment on the use of projection is in the work *De architectura libri decem* [9] by Marcus Vitruvius Pollio (Fig. 3)[2]. He described the three ways of projections – a top orthogonal view (ichnographia), a front orthogonal view (orthographia) and a view similar to perspective (scenographia). He believed that the views are created on the basis of experience and our vision.

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[1] The Philae island was an island in the river Nile (Egypt). Now is the island at the bottom of the Aswan dam.

[2] Marcus Vitruvius Pollio was a Roman architect. He lived in the first century BC.

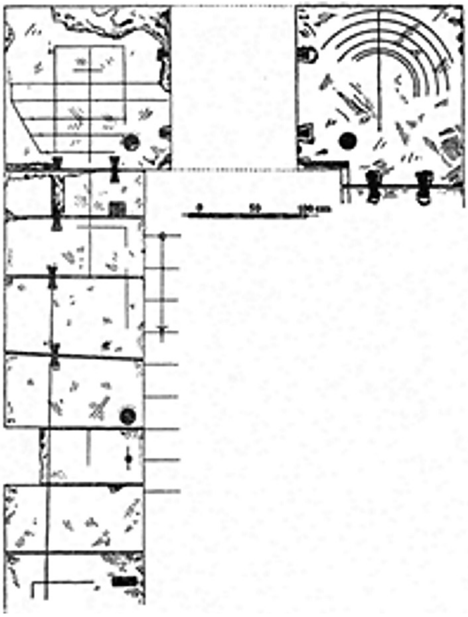

Fig. 2. Column carved into a stone on the Philae island [4]

M. VITRVVII POLLIONIS
DE ARCHITECTVRA
LIBRI DECEM,
CVM COMMENTARIIS
DANIELIS BARBARI,
ELECTI PATRIARCHAE
AQVILEIENSIS:
MVLTIS AEDIFICIORVM, HOROLOGIORVM,
ET MACHINARVM DESCRIPTIONIBVS,
CVM PRIVILEGIIS.
VENETIIS,
Apud Franciscum Franciscium Senensem, & Ioan. Crugher Germanum.
M. D. LXVII.

Fig. 3. Title page of Vitruvius's work [9]

2.2. Attempts at perspective

We cannot talk about the use of perspective in the Antiquity. On the other hand, in some wall paintings (Fig. 4) and frescos from this period we can see some attempts at perspective. No rules of perspective are observed; there are many vanishing points for one direction of lines and other mistakes. The characteristics of perspective were used accidentally with purpose to make the illusion of space more real.

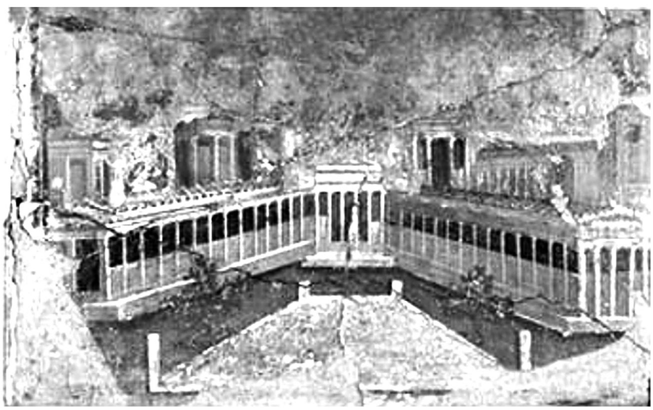

Fig. 4. Wall painting from the ancient city Pompeii created before 79 AD

3. Period of the Late Middle Ages and perspective in Renaissance

The long period of Middle Ages is difficult to investigate. We have few materials from the time before the 13th century to research the development of projection. The most of extant interesting drawings with elements of orthogonal projection were created between 1200 and 1500. The period of Renaissance was the main age of the development and the improvement of perspective because of the works of Renaissance artists.

3.1. Orthogonal projection

Temples or towers were a main topic of drawings in orthogonal projection. Typical feature was the use of arbitrary connection of the top and the front orthogonal view in one picture (Fig. 5)[3], but these two views were not always parts of one object. For example in the plan of the St. Vitus Cathedral in Prague (Fig. 6) by Peter Parler[4] we can see the front orthogonal view of the supporting system and the top orthogonal view of some temple tower.

Fig. 5. Sketch drawn by Villard de Honnecourt in about 1250

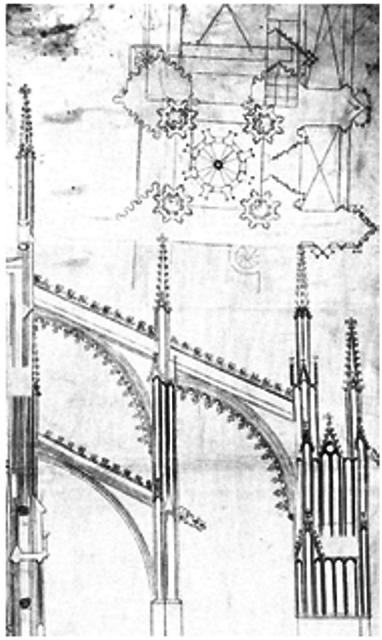

Fig. 6. Plan of the St. Vitus Cathedral from the 2nd half of the 14th centrury

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[3] A sketch in Fig. 5 is from the Honnecourt's sketchbook, which is available on <http://classes.bnf.fr/villard/feuillet>.

[4] The plans of the St. Vitus cathedral are deposited in the Library of the Viennese Academy of Fine Arts.

3.2. Witelo's work on geometry

The first important Polish geometer was Witelo[5]. He wrote a magnific work *Perspectiva*[6]. This work has ten books. Only the first of them, which contains sixteen definitions, five postulates and 137 propositions, has mathematical character. The book is structured upon the Euclid's *Element*. In the other books Witelo used the mathematics rules stated in the first one. The main topic of Witelo's work was optics, but hand in hand with optics he described many geometric rules connected with central projection[7].

3.3. Improvement of perspective

In the 14th century painters still used perspective in their paintings intuitively, as we can see in a painting by Giotto[8] (Fig. 7). However, many artists made effort to use and describe rules of perspective. Their aim was real illustration of space, as good as possible.

Fig. 7. Intuitive use of perspective in Giotto's painting from*Scenes from the life of Saint Francis* situated in the Bardi chapel in Florence

During the 15th century Renaissance artists used the correct principles of perspective more frequently[9]. Some famous artists were Filippo Brunelleschi (1377–1446), Piero della Francesca (about 1415–1492), Masaccio (1401–1428), Paolo Ucello (1397–1475) or Leon Battista Alberti (1404–1472). Flawless perspective can be found e.g. in the sketch to the painting *Adoration of the Magi* by Leonardo da Vinci[10] (Fig. 8) or in some paintings by Raffaello Sanzio (1483–1520).

[5] Witelo (about 1230–1280) was born probably in Legnica or Wroclaw (or in some surrounding village). He was a son of Turin's colonist and Polish women, because he called himself "filius Thuringorum et Polonorum". About his life we have only little information.

[6] This work was written between 1270–1278. It was firstly published in 1572 in Basel.

[7] For more about Witelo and his work see [12] or [16].

[8] Ambrogio di Bondone (about 1266–1337), known simply as Giotto, was an Italian painter and architect, one of predecessors of the Italian Renaissance.

[9] For more about the use of perspective in art see [3].

[10] Leonardo da Vinci (1452–1519) was an Italian Renaissance painter, architect, inventor, anatomist and writer.

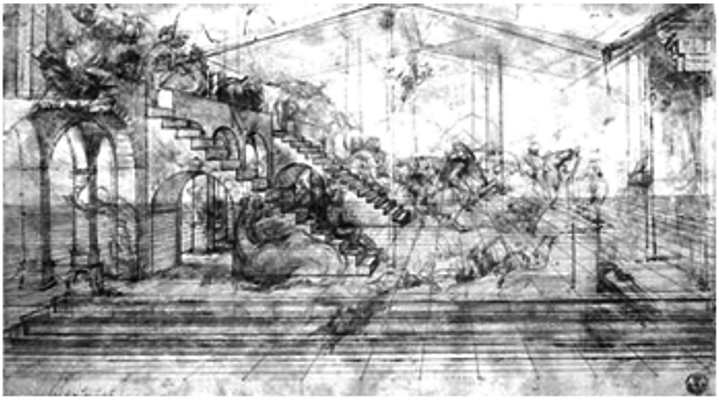

Fig. 8. Sketch to the painting by Leonardo da Vinci

In 1600 Guidobaldo del Monte[11] formulated the basic theorem of perspective, the projections of parallel lines meet at one point (the vanishing point of the parallel lines), in his work *Perspectivae libri sex* [7] (Fig. 9).

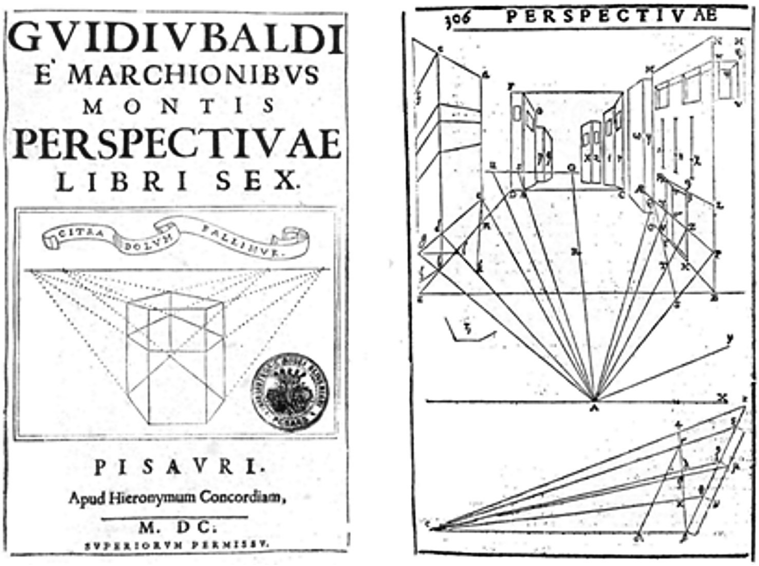

Fig. 9. Title page and one page from the work about perspective by Guidobaldo del Monte

Thanks to Renaissance artists the theory of perspective was concluded at the beginning of the 17th century. Of course, in the subsequent works on perspective constructions were improved and simplified.

[11] Guidobaldo del Monte (1545–1607) was an Italian painter, philosopher and astronomer.

4. Use of parallel projection from the 16th to the 18th century

It is typical for drawings created in the 16th century and later to be drawn according to rules of projections, and they are mainly unequivocal. Moreover, some works created in this period included not only correct illustrations, but also general rules of parallel projection. Therefore, we can call their authors the predecessors of Gaspard Monge.

4.1. Dürer's work on geometry

Some illustrations which look like constructed in Monge's projection can be found in *Underweysung der Messung mit dem Zirckel und Richtscheyt in Linien, Ebenen und gantzen Corporen*[12] by Albrecht Dürer[13]. He used orthogonal projection for construction of a cube in five different concrete positions in space (Fig. 10). Dürer started with one simple position (A) and then, using the rotation and symmetry in space, he derived the other positions of the cube (B–E) from the first one.

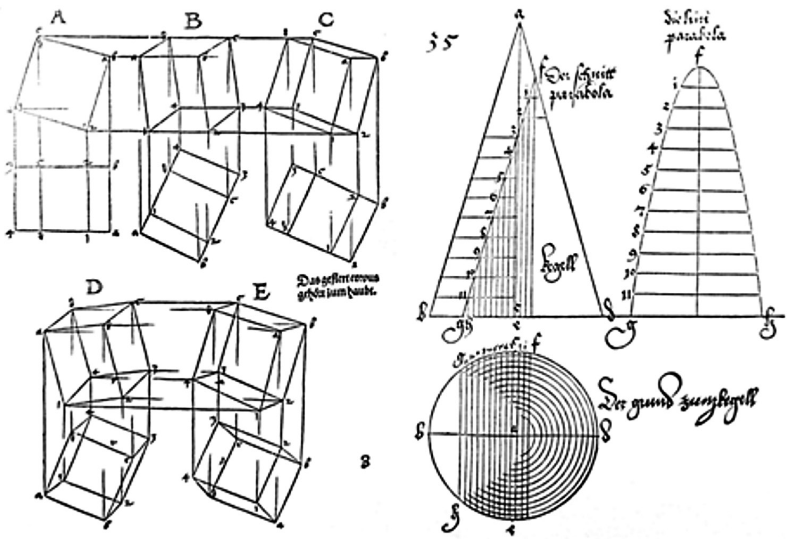

Fig. 10. Dürer's construction of different positions of a cube and construction of parabola

Moreover, Dürer used orthogonal projection and sections of a cone for construction of ellipse, parabola (Fig. 10) and hyperbola. He always chose suitable plane of section and constructed the top and the front view of the section 'point after point'. In this way he obtained

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[12] This book was the first big book on geometry in German. It was published in two editions – 1525 and 1538 in Nürnberg. The text is available on <http://digital.slub-dresden.de/werkansicht/dlf/17139/1/cache.off>.

[13] Albrecht Dürer (1471–1528) was a German painter and a graphic artist with interest in mathematics, especially geometry. He researched the use of geometry in art; he was primarily interested in the theory of proportions of human body.

enough information for constructing the conic in real size and he presented the construction on the right side of each page.

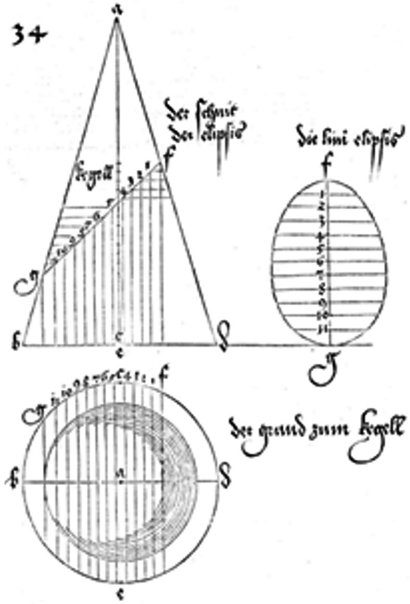

Fig. 11. Mistake in Dürer's construction of ellipse

Dürer's method of the construction of conics is correct, but in his drawing of ellipse he made one mistake. He was probably convinced that the section should be an oval curve with only one axis of symmetry (like an egg) and he constructed it so inaccurately that he really obtained what he expected (Fig. 11).

4.2. Cavalier perspective

For an easy projection of forts (Fig. 13), city plans (Fig. 14), etc. a new method of projection, called Cavalier projection[14], emerged about 1600. It is a special type of oblique projection in which the top view of a building is not distorted in the same way as the height of the building, because the plane of projection is chosen parallel with the ground of the building and the direction of projection is 45° (Fig. 12).

[14] Cavalier projection is sometimes called Cavalier perspective or military perspective. But these names do not refer to perspective as a kind of central projection.

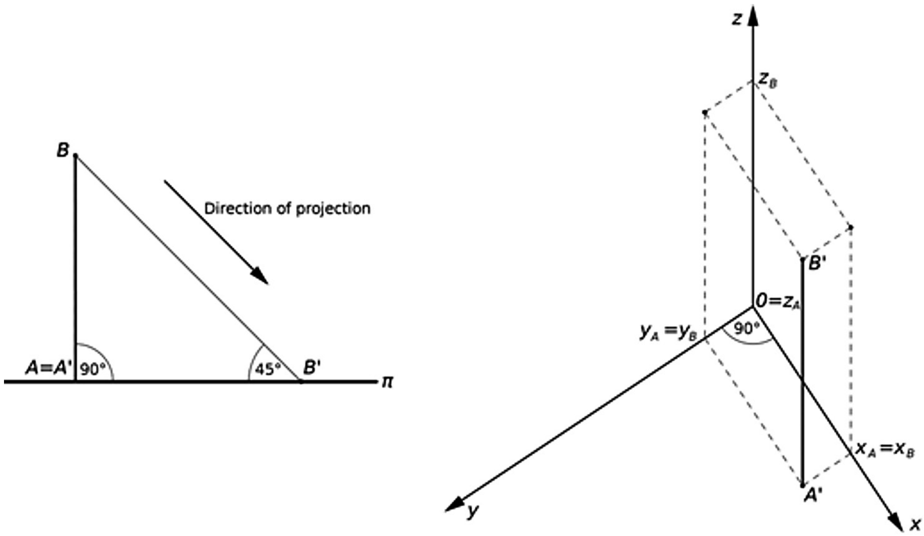

Fig. 12 Principle of Cavalier projection

Fig. 13. Plan of a fort by Jacques Perret from 1602 [4]

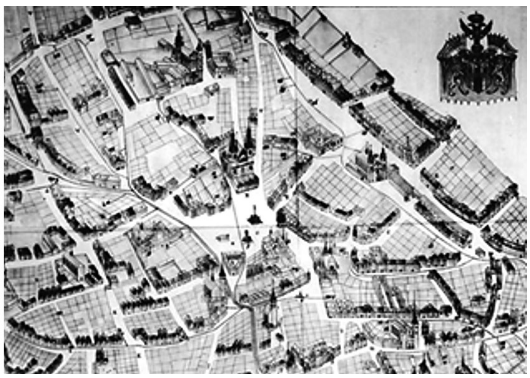

Fig. 14. Plan of a water pipeline by J.V. Veselý from 1720 [14]

4.3. Stereotomy

In essence, Dürer used Monge's projection. Similarly, in many drawings from the period between 16th and 18th century we can find the top and the front view like in Monge's projection (Fig. 15), but these illustrations have common characteristic – they were used only for unequivocal projection of some space object into plane, not for solving of some space problem in plane.

Attempts to generalize the rules of orthogonal projection and use it for solving of space problems originated in works on stereotomy (the theory about cutting stone and wood). The first works on this topic were *Le premier tome de l'architecture* (Paris, 1567) by Philibert de l'Orme[15], *Les secrets de l'architecture* (La Flèche, 1642) by Mathurin Jousse[16] (Fig. 16) or *Brouillon project d'une exemple d'une maniere universelle du S. G. D. L. touchant la pratique du trait à preuves pour la coupe des pierres en l'architecture* (Paris, 1640) by Girard Desargues[17].

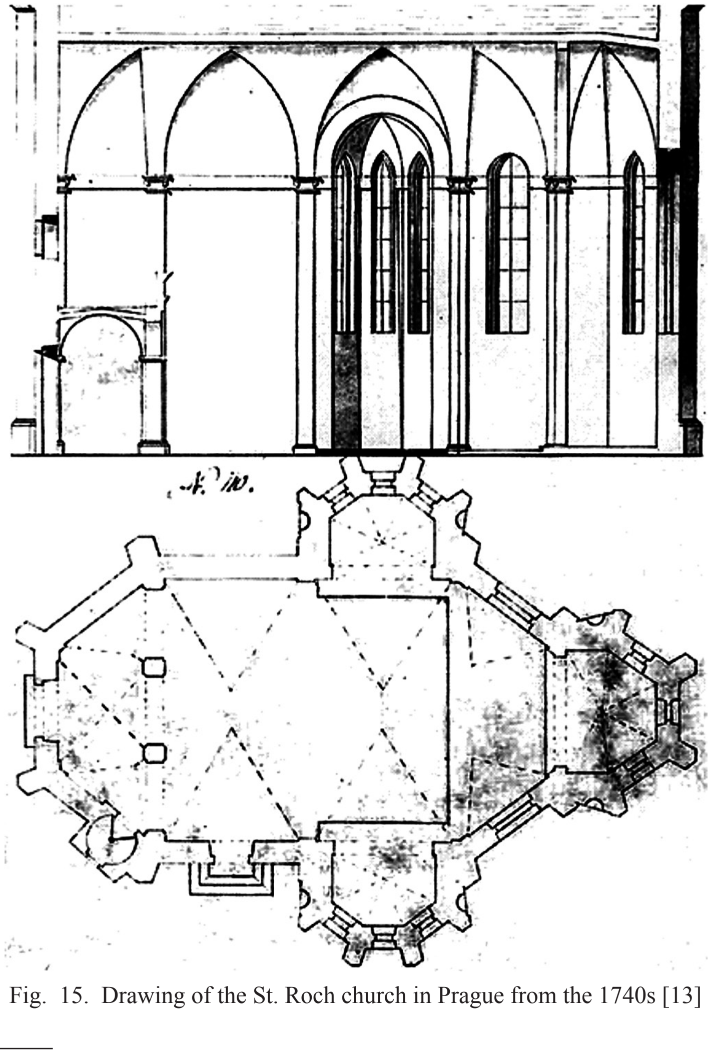

Fig. 15. Drawing of the St. Roch church in Prague from the 1740s [13]

[15] Philibert de l'Orme (about 1514–1570) was a French architect. His first significant work in architecture was the design of the castle *Château de Saint-Maur* in Paris.

[16] Mathurin Jousse (1575–1645) was a French architect. He was the author of many works on building trades (joinery, carpentry, locksmithery, etc).

[17] Girard Desargues Lyonnais (1591–1661) was a French mathematician, architect and engineer. He was interested in perspective, stereotomy and conics. He introduced improper elements and polarity into geometry. He was one of founders of projective geometry.

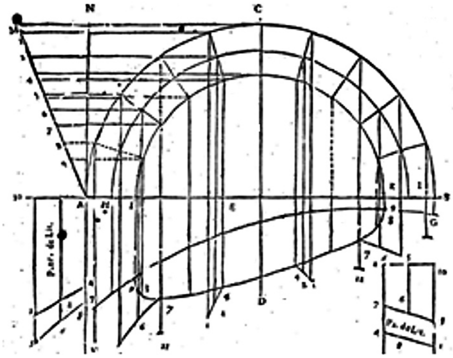

Fig. 16. Orthogonal projection of correct laying of stones under an oval entrance from Jousse's work

LA THEORIE ET LA PRATIQUE
DE LA
COUPE DES PIERRES
ET DES BOIS
POUR LA CONSTRUCTION DES VOUTES
OU
TRAITÉ DE STEREOTOMIE
A L'USAGE DE L'ARCHITECTURE.
TOME SECOND

A STRASBOURG
A PARIS,
M DCC XXXVIII.

Fig. 17. Title page of Frézier's work

A significant work on stereotomy was *La théorie et la pratique de la coupe des pierres et des bois, pour la construction des voutes et autre parties dés bâtimens civils at militaires, ou Traité de stéréotomie à l'usage de l'architecture* (Strasbourg, 1737) (Fig. 17) by Amédée François Frézier[18]. In the first part Frézier described the general principles of orthogonal projection. In the other parts he introduced the applications of projection in stereotomy. This work was probably the most important work on projection before the publication of the Monge's work on descriptive geometry.

[18] Amédée François Frézier (1682–1773) was a French officer, engineer and mathematician. In 1752 he became a member of the French Academy of Sciences.

5. Gaspard Monge and his *Géométrie descriptive*

Gaspard Monge[19] taught a new method of geometry in which he used construction instead of plaster models in Mézières from 1766. However, this teaching was forbidden because of the secrecy of the new method of geometry for military purposes.

After 1794 (during the French Revolution) political situation in France was changed and Monge could teach his new method of geometry, which he called descriptive geometry, at École Normale and École Polytechnique in Paris.

In the school year 1794/1795 Monge published the first edition of his work on descriptive geometry with title *Textes des leçons de géométrie descriptive données à l'École Normale* in the school journal *Séances des Écoles Normales*. In this work descriptive geometry was firstly conceived as a science, therefore Monge is usually called the founder of descriptive geometry.

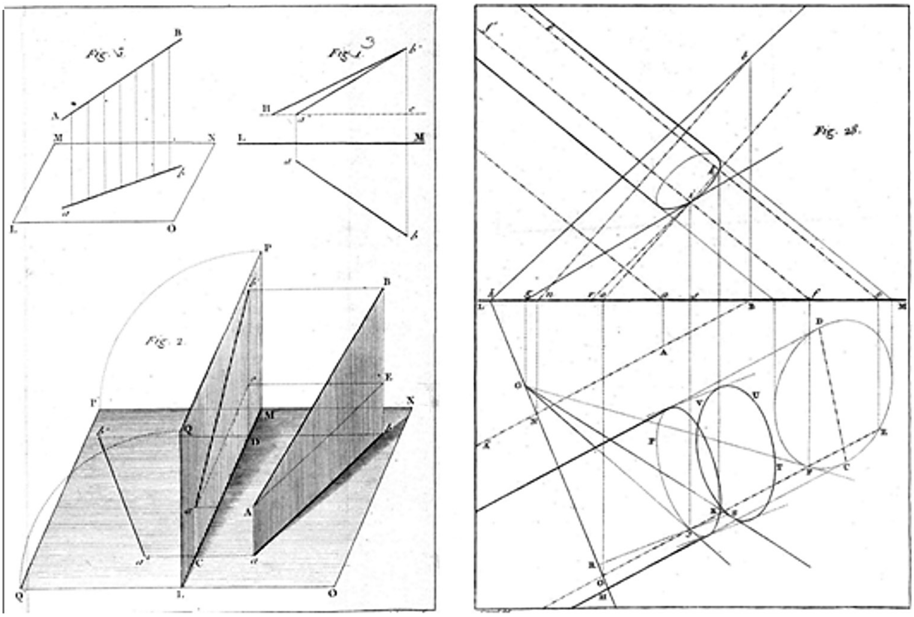

Fig. 18. Illustrations from Monge's work *Géométrie descriptive*

In the 1799 the second edition of Monge's work was published as a book with the title *Géométrie descriptive. Leçons donées aux Écoles Normales, l'an 3 de la Republique* (Fig. 18). The book has five parts. Three of them are about theory and general methods of descriptive

[19] Gaspard Monge (1746–1818) was a French mathematician and physicist. When he was twenty two years old, he became a professor of mathematics at school in Mézières. During the French Revolution he was called at École Normale. After establishment of École Polytechnique Monge was suggested as the president of this school, he refused this position, but taught stereotomy, descriptive geometry and physics here till 1809. For more information about Monge's work and life see [6, 8, 11].

geometry; in the fourth part there are applications of these methods for sections of curved surfaces and in the last part there is theory of curves and surfaces with use of differential geometry.

Monge's work on descriptive geometry was published repeatedly in French. Starting from the fourth edition (1820), it was supplemented with parts *Théorie des ombres* and *Théorie de la perspective* by Barnabé Brisson[20].

5.1. Translations of Géométrie descriptive

Monge's *Géométrie descriptive* was translated into many languages. The book was published in Spanish (1803), English (1809 and 1851), Italian (1838), German (1900) and in Russian (1947). At present, reprints of these editions are generally available[21].

5.2. Monge's students and successors

Other important works on descriptive geometry were published in the first half of the 19th century by Monge's students S.F. Lacroix[22] and J.N.P. Hachette[23]. Another important professor of descriptive geometry at École Polytechnique was Charles François Antoinne Leroy (1780–1854). He wrote *Traité de géométrie descriptive* (Paris, 1834). This work was published fifteen times before 1910 and was translated into German.

All these works (together with Monge's work) and some others influenced the development of descriptive geometry in other countries in Europe.

6. Boom of descriptive geometry in Cisleithania

The development of industry in Europe in the first half of the 19th century caused the development of technical sciences and education during the 19th century. Descriptive geometry gradually began one of obligatory subjects of technical studies. The biggest boom of descriptive geometry came (except of France) in Germany[24], Italy[25], Great Britain[26] and in Austro-Hungarian Empire, primarily in Cisleithania.

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[20] Barnabé Brisson (1777–1828) was one of Monge's students at École Polytechnique. After completing his studies he became a building ingeneer, in practice he applied descriptive geometry to building of navigation channels. In 1808 he married Monge's niece Anne-Constance Huart de l'Enclose.

[21] Many reprints are available e.g. on <www.amazon.com>.

[22] Sylvestre François Lacroix (1765–1843) was a French mathematician. From 1794 he helped his teacher Monge with preparing materials for lessons on descriptive geometry. He wrote a work *Essai d'géométrie sur les plans et les surfaces courbes* (Paris, 1795), which was published repeatedly with the title *Complément des élémens de géométrie.*

[23] Jean Nicolas Pierre Hachette (1769–1834) was a French mathematician. He became a successor to Monge at École Normale. He extended the Monge's work on descriptive geometry with two addenda *Suppléments à la Géométrie descriptive de Monge* (1811, 1818).

[24] Among known German geometers we can mention Karl-Wilhelm Pohlke (1810–1876), Guido Schreiber (1799–1871), Bernhard Gugler (1812–1880) or Christian Wiener (1826–1896).

[25] Among known Italian geometers we can mention Vincenzo Flauti (1782–1863), Giusto Bellavitis (1803–1880) or Gino Benedetto Loria (1862–1954).

[26] The axonometric projection has an origin in Great Britain. Regarding this we can mention William Farish (1759–1837) or Peter Nicholson (1765–1844); for more see [5].

In the second half of the 19th century and at the beginning of the 20th century new works and textbooks on descriptive geometry were written and methods of projections (mainly axonometric projections, theory of shadows or photogrammetry) were improved.

6.1. Descriptive geometry teaching at secondary schools

In 1849 (Exner-Bonitz reform of secondary schools in Cisleithania) a new modern type of secondary schools with emphasis on natural sciences and modern languages was established and was called ‘Realschule’[27]. At these schools descriptive geometry has been taught since the 1850s, because students of Realschules often continued their studies at polytechnics and therefore they needed to have knowledge of descriptive geometry.

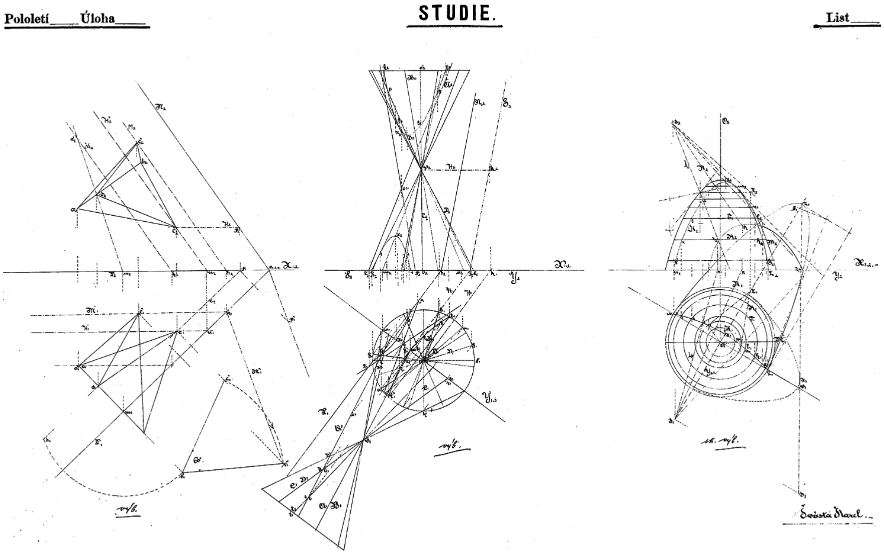

Fig. 19. School-leaving exam work from Czech Realschule in Hradec Králové worked out by Karel Švásta in 1877

The school-leaving exam in descriptive geometry was obligatory at Realschules since the 1870s. Students wrote a five-hour test with three or four exercises. They had to construct them in ink (Fig. 19)[28] and describe the process of the solution. In comparison with the present time the exams were very difficult. Current students of descriptive geometry at universities would probably have problems with similar exercises.

[27] We will use the term ‘Realschule’ (from German) as singular and ‘Realschules’ as plural.

[28] In the Figure 19 there are drawings of this exercises [15]:

1. Orthogonal projections of a triangle *abc* are given; rotate it on the side *ab* about 60°.
2. Draw a hyperbolic section of a cone surface of revolution with a directing circle in the first plane of projection. The plane of section is in general position.
3. Draw the shadow of a parabolic solid whose axis is perpendicular to the first plane of projection. The rays come out from one point.

For secondary school students new textbooks and collections of exercises on descriptive geometry were also written. One of them, the Czech book *Deskriptivní geometrie pro střední školy reálné* (Prague, the first edition: 1875–1877, the other editions: 1887, 1893, 1900, 1905) by Vincenc Jarolímek[29] was translated into Bulgarian. Bulgarian edition was published in 1895 in Plovdiv (Fig. 20). This action was only one of many attempts to spread descriptive geometry to countries of Eastern Europe[30].

6.2. Lectures on descriptive geometry at polytechnics and universities

Descriptive geometry has been taught at polytechnics in Cisleithania[31] from the first half of the 19th century (e.g. at Prague Polytechnic School since the 1830s). At some of them the professorship of descriptive geometry was established (e.g. in Vienna in 1842, in Prague and Brno in 1850, in Graz in 1861)[32].

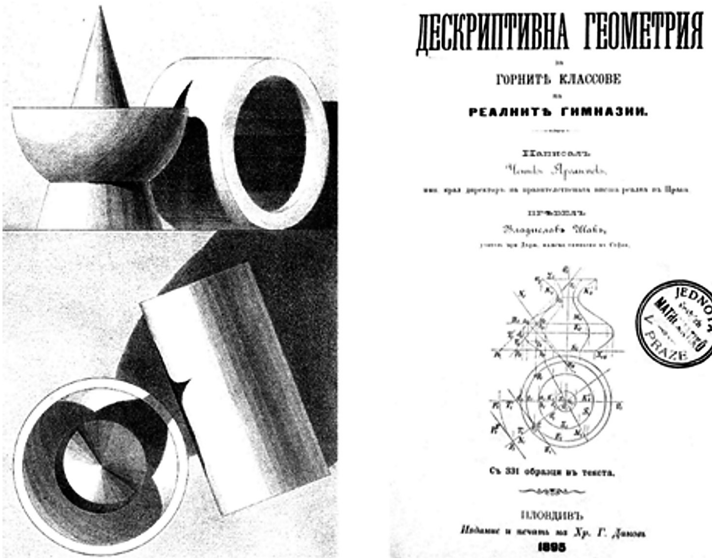

Fig. 20. Illustration from the first edition of Jarolimek's secondary textbook and the title page from Jarolimek's textbook translated into Bulgarian

[29] Vincenc Jarolímek (1846–1921) was a teacher of mathematics and descriptive geometry at Realschules. In 1907 he became a professor of descriptive geometry at Prague Polytechnic School. For more about the first Czech textbooks on descriptive geometry see [2].

[30] Regarding influence of Czech mathematics in Bulgaria see [1].

[31] Polytechnics (Fig. 21) were established in Prague (1806), Graz (1811), Vienna (1815), Brno (1850) and Lemberk (now Lviv, 1871). Similar schools were established also in other countries of Europe, the firsts of them were in Italy (Neapol, Roma), Germany (Berlin, Karlsruhe, München, Dresden), Great Britain (London), etc. All polytechnics were established following the model of École Polytechnique.

[32] For more about systemization of descriptive geometry professorships at technical schools in Cisleithania see [10].

A syllabus of descriptive geometry at Prague Polytechnic in 1852 included various kinds of methods of projections (orthogonal, oblique, perspective), theory of curves and surfaces and other topics. The lectures were provided by Rudolf Skuherský[33], who lectured according to Hönig's[34] work and his own works. Number of lessons a week was high – about 12 hours. The lectures were obligatory for students of building constructions at first, later for students of engineering, architecture, forestry and other areas of study as well.

At the other polytechnics the syllabi of descriptive geometry were similar. Sometimes extra lectures on projective geometry, perspective or stereotomy were provided. The quality of lectures depended mainly on the lecturer, but we can say that it was generally very high. Besides Hönig's textbook the above-mentioned work by F.A. Leroy was often used. In the second half of the 19th century and in the first third of the 20th century new scientific works and textbooks on descriptive geometry for polytechnic students were published. Students also prepared litographed notes of some lectures.

Fig. 21. Map of polytechnics in Cisleithania

A growing number of polytechnic students caused growing necessity for teachers of descriptive geometry not only at polytechnics but also at secondary schools. These teachers graduated from polytechnics at first, but studying at a university was a better way of preparation for teaching career. For example, the future teachers of descriptive geometry

[33] Rudolf Skuherský (1828–1863) studied at Prague Polytechnic School and at Viennese Polytechnic School. In Vienna he was the student of the professor Johann Hönig. In 1854 he became the first professor of descriptive geometry at Prague Polytechnic School.

[34] Johan Hönig (1810–1886) was a professor of descriptive geometry at Viennese Polytechnic School between 1843–1870. He wrote a textbook on descriptive geometry *Anleitung zum Studium der darstellenden Geometrie* (Wien, 1845). This book was used by students of polytechnics in Cisleithania for many years.

could study this subject at Czech University in Prague from the 1910s courtesy of Jan Sobotka. At German University in Prague students could attend periodical lectures on descriptive geometry only a few years later.

6.3. Next personalities of descriptive geometry in Cisleithania

Regarding the textbooks and lectures on descriptive geometry we mentioned only a few personalities who made descriptive geometry in the Central Europe famous. But there were many other persons who contributed to the development of descriptive geometry. Let us recall their names at least: František Tilšer (1825–1913), Čeněk Hausmann (1826––1896), Gustav Adolf Viktor Peschka (1830–1903), Josef Schlesinger (1831–1901), Rudolf Niemtschik (1831–1876), Wilhelm Otto Fiedler (1831–1912), Rudolf Staudigl (1838–1891), Emil Koutný (1843–1880), Karel Pelz (1845–1908)), Emil Müller (1861–1927), František Kadeřávek (1885–1961) and others. For more about their work and life see [6] or [10].

7. Conclusion

The article showed a short summary of history of descriptive geometry and its coming to the Central Europe. In the 19th century and in the first half of the 20th century this geometry together with projective geometry was at its peak. In technical literature we can find in this connection the terms "Czech geometrical school" or "Viennese geometrical school".

This trend lasted until the World War II. Afterwards descriptive geometry never had such a big role in education. Currently computer programs are usually used for constructions of objects in geometry and only few of people with high knowledge of descriptive geometry are needed as employees in industry or as teachers.

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