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REMARKS ON NON-EUCLIDEAN GEOMETRY IN THE AUSTRO-HUNGARIAN EMPIRE

UWAGI O GEOMETRII NIEEUKLIDESOWEJ W MONARCHII AUSTRO-WĘGIERSKIEJ

A b s t r a c t

Since 1800s, Central European mathematicians have achieved great results in hyperbolic geometry. The paper is devoted to brief description of the background as well as history of these results.

Keywords: *Austro-Hungarian Empire*, *history of hyperbolic geometry*

S t r e s z c z e n i e

Od XIX w. matematycy w Europie Środkowo-Wschodniej osiągali znaczące wyniki w geometrii hiperbolicznej. Niniejszy artykuł zarysowuje tło i historię tych wyników.

Słowa kluczowe: *Monarchia Austro-Węgierska*, *historia geometrii hiperbolicznej*

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1. Introduction

The Bolyai geometry is an important historical phenomenon in mathematics, and a timely research topic with potential applications. I will say a few words about these topics here.

I would like to say first something about the expression "Bolyai geometry".

Officially the hyperbolic geometry is called B-L geometry, but this form is not really used anywhere. In 1894 Poincaré was the chairman of the committee that compiled the bibliography of hyperbolic geometry. The title was originally Lobachevsky's Geometrie. However, it was changed to Geometrie de Bolyai et Lobachevsky – as a result of Hungarian mathematicians' argumentations (see, e.g. [16]).

The most common name is "hyperbolic geometry"; sometimes "Bolyai-Lobachevsky-Gauss" is used. In the Russian-speaking world the common name is 'Lobachevski's geometry', while in Hungary it is called "Bolyai geometry". However, this is not only inaccurate, but can also be confusing, because "Bolyai geometry" in mathematics has a special meaning: it is the name of absolute geometry discovered by Bolyai János. Bolyai worked out a geometry where both the Euclidean and the hyperbolic geometry are possible, depending on a parameter *k*. Three elementary geometries exist: hyperbolic, parabolic and elliptic geometry. The names refer to the Greek names of the sum of the three angles of a triangle.

However, in this presentation I will call the hyperbolic geometry "Bolyai geometry", as we frequently do in Hungary.

The hyperbolic geometry is of importance in philosophy of mathematics, and also in mathematics education. According to Wikipedia (https://en.wikipedia.org/wiki/Non-Euclidean_geometry).

„The existence of non-Euclidean geometries impacted the ‹‹intellectual life›› of Victorian England in many ways and in particular was one of the leading factors that caused a re--examination of the teaching of geometry based on Euclid's Elements. This curriculum issue was hotly debated at the time and was even the subject of a play, Euclid and his Modern Rivals, written by Lewis Carroll, the author of Alice in Wonderland."

The hyperbolic geometry slowly entered the public consciousness; M.C. Escher's graphics played a large role in this process. Computer technology offers additional options. The model of the elliptical geometry is the spherical geometry. The hyperbolic geometry does not have a model in Euclidean space, hence the importance of the hyperbolic computer drawing programs. There have been teaching experiments concerning the role of hyperbolic geometry in the school curriculum (see [19]).

2. Backgrounds of hyperbolic geometry

Hyperbolic, or Bolyai, geometry has ancient antecedents. The oldest maps such as Hecateus indicate Earth to have the shape of a disc, in accordance with the tree-of-life picture of the myths. Our lives happen in a disc-shaped world lying at the foot of the tree of life.

According to the researches of Imre Tóth, the thought of possible multifarious geometry emerged already in the mind of the ancient Greek. The Euclidean geometry was built on the parallel axiom. This geometry, formulated by Euclid, was a choice among possible geometries [18]. This view disappeared later on, and the Euclidean geometry seemed to be only possible geometry. The parallel axiom seemed to be a theorem, which many mathematicians tried to prove. The attempts at an indirect proof did not lead to a paradox, so the possibility emerged that the negation of the statement could be true. We can read in detail about this procedure, and also about the activity of Saccheri and Lambert, e.g., in [13].

The multifarious geometry was discovered separately by Bolyai and Lobachevski. The question of who was first is very hard to decide because the writing of the manuscript, the first publications and the reaction of Gauss were a process in which Bolyai or Lobachevski alternately had priority. Gauss knew about both discoveries and recognized their significance, but he did not let them be published for the wider readership. The first studies about foundation of geometries came out after Gauss' death. The emphasis is now on the plural: there is not 'a geometry', but there are 'geometries'. The results of Bolyai and Lobachevski can be found in some subsequent works without references, e.g. in the great work of Riemann (see [12]).

But this theory was not generally appreciated. Beltrami thought that hyperbolic geometry was not an independent, new theory, but a part of differential geometry. Hyperbolic plane was a special kind of a surface with constant curvature. What he indicated was that J. Bolyai and N.I. Lobachevsky had not really introduced new concepts at all, and so there was no alternative to Euclidean geometry (see [2, 3]).

The turn of the previous century was the era of great development in Central and Eastern Europe.

The Compromise of 1867, which created the Dual Monarchy of Austria-Hungary, caused quick economic development and at the same time accelerated the Hungarian cultural development. Hungarian became the language of instruction from elementary schools to universities. (Earlier, it was first Latin, then German.) The Hungarian mathematical research was integrated into the international scientific world. Many articles written by Hungarian authors appeared: in Comptes Rendus nearly 20, in German journals 100 articles. The majority of the articles were first published in Hungarian. In that period two Hungarian mathematics journals were founded: *Mathematikai és Természettudományi Értesítő* (1882–1941) and *Mathematikai és Physikai Lapok,* (1891–1944).

Famous scholars were elected as the HAS (Hungarian Academy of Science, Magyar Tudományos Akadémia) members, for example Arthur Cayley, Charles Hermite, Hermann Helmholtz, Hugo Kronecker, Paul du Bois-Reymond, Felix Klein, Gaston Darboux and Gösta Mittag-Leffler.

3. The perception in the Austrian-Hungarian Monarchy

The international acknowledgement began in Göttingen, and French, Italian, American translations were completed afterwards. This process is well documented, we can read about it for example in [15, 9]. In Budapest András Benedek (see [5]) and János Tanács (see [17]), in Transylvania Tibor Weszely (see [20, 21]) work on this period, i.e., the end of 19th century and the beginning of 20th century in Hungary. Emil Molnár (see [10, 11]) investigates the history and the modern applications of hyperbolic geometry. I consulted their works while preparing this article. There are also earlier Hungarian books on mathematics history, mainly by Szénássy (see [15, 16]), which contain information about perception of Bolyai's geometry.

Baltzer's book titled "Elemente der Mathematik" (1860) was the first university course book, and a popular one, which mentioned some results of the two Bolyais. The name of Bolyai became known after Gauss' death, when his legacy, including his correspondence, was analysed. In 1867 Hoüel asked for the Appendix from Cluj Napoca. He got a printed copy presumably, and he had it published in French: János Bolyai, *La science absolute de l'espace*,1867, Bordeaux. Hoüel [8] translated it (see [16]). Almost at the same time (1869) an Italian historian of mathematics also asked for it from Boncompagni.

And what happened in Austria, Hungary and the neighbouring countries? The following information, gathered thanks to many colleagues at conferences and by Internet, comes from my lecture given at the International Congress of History of Science and Technology in Budapest, 2009 [1].

Frischauf held a lecture in Graz about non-Euclidean geometry in 1871/72 and the material of the lecture was published, as well. Frischauf: *Absolute Geometrie nach Johann Bolyai*, Leipzig, 1872 (see [16]).

Until 1900 almost nothing about non-Euclidean elementary geometry was taught, except some differential geometry, theory of surfaces, projective geometry and spherical trigonometry.

Gustav Kohn's lecture was the first on non-Euclidean geometry in 1905 (G. Kohn was in Berlin as a „student" of Otto Stolz, 1870–1871)[1].

The hyperbolic geometry came to Prague from Russia.

Eduard Weyr (1852–1903) was the first Czech professor of mathematics who wrote on the non-Euclidean geometry in the Czech lands. In 1896, he published two short articles which gave account of Lobachevsky's centenary celebration in Russia and which contained the first analysis of his works in Czech. Eduard Weyr translated some interesting and important parts from the proceedings which were published by University in Kazan (i.e. the parts form the lectures of F. M. Suvorov (1845–1911) and A.V. Vasiljev (1853––1929)). See [6, 7, 22–25][2].

University of Belgrade was established in 1905. Until 1946 there were no lectures on geometry. Research and lectures on hyperbolic geometry started after 1946[3].

There is a famous school of differential geometry in Belgrade, working on surfaces of constant curvature.

First seminar on Bolyai geometry was in Kolozsvár -Cluj

Gyula Vályi (1855–1913) was a mathematician at the University of Kolozsvár. He held a course on Bolyai geometry in 1891–1892.

What were the origins of his seminar?

The scientific source: Vályi saw the role of new theories of geometry in contemporary mathematics during his scholarship in Berlin, 1878–1880.

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1 Thanks to Hellmuth Stachel and Christa Binder for references.

2 Thanks to Martina Bečvářová for numerous references.

3 Thanks to Mileva Pranovic for this information.

The personal source: He had a copy of Tentamen (1. edition, 1832), dedicated by Farkas Bolyai to his father, Károly Vályi, who was a student of Farkas Bolyai. This book was available neither in the libraries, nor in the book shops. Luckily the Tentamen, the book with Appendix was preserved as a relic by the Vályi family. (We know all this from a personal letter of a university professor of the name of Réthy Mór)[4].

There was a chain of teachers and their students between Bolyai and Szénássy, who was a great Hungarian historian of mathematics. David Lajos (University professor of mathematics in Kolozsvar and in Debrecen), was a student of Gyula Vályi, and Barna Szénássy (University professor of mathematics in Debrecen) was the student of David Lajos. This chain explains the mystery how the information was transmitted when neither the book nor the manuscript was available.

The research on history of mathematics started early, but the non-Euclidean geometries became a part of university curriculum only later. Only in 1930s did Béla Kerékjártó write his books. The Foundations of Geometry, Foudation of Projective geometry, 1937, 1944.

4. Modern applications

It is possible that crystallography can be expressed more easily with non-Euclidean than Euclidean geometry. There are a lot of articles by Emil Molnár, some of which are intended for secondary mathematics teachers (see [10]).

A new type of the Internet browser was built on a hyperbolic tree (see [14]). We can read on this topic in the broader context of dynamic visualization and hyperbolic mappings.

5. Conclusion

Since 1800s, Central European mathematicians have achieved great results in hyperbolic geometry. However, these achievements (and other elements of modern mathematics) are still absent from the school curricula. This presents a challenge for mathematics education.

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