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## STRENGTH HYPOTHESES – A NEVER ENDING STORY

### HIPOTEZY WYTRZYMAŁOŚCI – NIEKOŃCZĄCA SIĘ OPOWIEŚĆ

#### Abstract

Up to now there are a lot of discussions concerning criteria of the limit state of a material. The classical criteria like the Huber-von Mises-Hencky criterion are mostly used by the engineers. With the creation of new materials with non-classical behavior (different limit values at tension and compression, compressibility, Poynting-Swift effect, etc.) the classical criteria fail. It will be shown how new criteria can be created by some unified approach.

*Keywords: generalized strength criteria, equivalent stress invariants, materials with non-classical behavior*

#### Streszczenie

Liczne dyskusje dotyczące kryteriów osiągnięcia przez materiał stanów granicznych trwają do dzisiaj. Najczęściej stosowanymi przez inżynierów są takie klasyczne kryteria, jak kryterium Hubera-von Misesa-Hencky'ego. Wraz z powstaniem nowych materiałów, zachowujących się w sposób nieklasyczny (różne wartości graniczne po stronie rozciągania i ściskania, ściśliwość, efekt Poyntinga-Swifta etc.), dotychczasowe klasyczne kryteria stają się niewystarczające. W artykule pokazano, jak stworzyć nowe kryteria za pomocą zunifikowanego podejścia.

*Słowa kluczowe: uogólnione kryteria wytrzymałościowe, zastępcze niezmienniki naprężenia, materiały zachowujące się nieklasycznie*

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## 1. Motivation

In 1982 Michał Życzkowski<sup>1</sup> has published his famous monograph *Combined Loading in the Theory of Plasticity* [27]. The main features of this book are the plasticity as a limit state of the material behavior, multi-axial loading conditions resulting in the necessity to present the material behavior by three-dimensional constitutive equations, the introduction of an equivalent stress for comparison purposes with the measurements from tests, the dependence of the equivalent stress on stress invariants, etc. This paper is devoted to the question how the equivalent stress expressions can be formulated. This item will be discussed with respect to the necessity to introduce criteria of the limit states which are based on the equivalent stress and further characteristics (parameters) of the material behavior.

## 2. Introduction

The need of criteria to describe the limit behavior of materials (transition from the elastic to the plastic range, loss of stiffness among others) is obvious. As usual one has multi-axial material behavior under real loading cases, i.e. a three-dimensional stress and strain state must be considered. At the same time mostly single specific properties of the material like the yield stress or the ultimate strength are obtained in tests and the question arises how to compare the multi-axial behavior expressed for example by the stress tensor  $\sigma$  with the scalar-valued material properties.

The introduction of the equivalent stress is a typical engineering approach since there are no physical principles like the balance equations in *Continuum Mechanics* [12, 16, 25] for comparison purposes. On the other hand, it should be compared scalar variables from tests with the equivalent stress  $\sigma_{eq}$  which is a function of the stress tensor and parameters related to some characteristics of the material behavior. The equivalent stress must be estimated for any arbitrary stress state and compared with a uniaxial state, for example the tension state. Let us introduce the material parameter  $\sigma_+$  as a parameter of the limit state established in the tension test. In this case the equivalent stress should be related to this material parameter by

$$\sigma_{eq} < \sigma_+$$

If  $\sigma_{eq}$  is less than  $\sigma_+$  the structure or the structural element can be exploited safe otherwise a failure or limit state occurs. In addition, in the engineering practice some safety coefficients are introduced. This special item is not in the focus of this paper.

Examples of classical hypotheses, presenting the material behavior at the limit state are based on the normal, the shear stresses or the distortion energy. The following hypotheses are mostly presented in textbooks on *Strength of Materials* or *Mechanics of Materials*: the maximum of the normal stresses, the maximum of the shear stresses and the maximum of the

<sup>1</sup> Prof. Życzkowski (1930–2006) was up to the end of his life an active member of the GAMM (International Association of Applied Mathematics and Mechanics) and the Editorial Board of the ZAMM (Journal of Applied Mathematics and Mechanics), which was founded in 1921 by Richard von Mises. He has organized together with his colleague G. Szefer the Annual Conference of the GAMM in 1991 in Krakow, which was the first Annual Conference in Poland.

distorsion energy. They are formulated as criteria containing information on the stress state and one material parameter, established, for example, in the tension or the torsion test.

In the case of arbitrary stress states the development of strength criteria should be coincide with images about the three-dimensional material behavior. The comparison of arbitrary stress states with material properties from one-dimensional tests is not possible, if one realizes only one type of tests. In addition, the arbitrary stress state is characterized by a tensor-valued variable, from the tests one gets usually scalar-valued characteristics. Since the mathematical type of the variables is different (scalars, second rank tensor), a direct comparison is impossible.

The first improvements of one-parameter-criteria, which are based on the use of experimental data from one type of tests, known from the 19<sup>th</sup> century. This development is characterized by the formulation of phenomenological criteria including beside the reference property  $\sigma_{eq}$  additional parameters. Examples are the maximum strain criterion suggested by Mariotte, St. Venant among others (with the Poisson's ratio as an additional parameter), the Mohr-Coulomb criterion and the Drucker-Prager criterion. Later criteria with more than two parameters were introduced (see, for example, [1] and the references within). These and other criteria approximate the given experimental data and extrapolate the experiences in the whole range of validity. An "exact" solution cannot be presented in general since one has often a problem with insufficient and/or infinite input information.

### 3. Classical Formulation – Isotropic Behavior

Let us suggest the following general form of strength criteria

$$\Phi(\boldsymbol{\sigma}, \sigma_{eq}) = 0$$

In addition, let us assume isotropic material behavior. In this case  $\sigma_{eq}$  is a function of three independent invariants of the stress tensor only. In the literature various sets of invariants are used [1, 6, 10, 27] among others. For example, in [27] the following invariants are discussed:

– the **basic invariants**

$$I_{1\sigma} = \boldsymbol{\sigma} \cdot \boldsymbol{I}, \quad I_{2\sigma} = \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}, \quad I_{3\sigma} = (\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}) \cdot \boldsymbol{\sigma}, \quad (1)$$

– the **principal invariants** following from the eigenvalue problem

$$(\boldsymbol{\sigma} - \lambda \boldsymbol{I}) \cdot \boldsymbol{n} = \mathbf{0}, \quad (2)$$

which results the characteristic equation

$$\lambda^3 - J_{1\sigma} \lambda^2 + J_{2\sigma} \lambda - J_{3\sigma} = 0 \quad (3)$$

with the invariants [19]

$$I_{1\sigma} = \text{tr} \boldsymbol{\sigma} = \boldsymbol{\sigma} \cdot \boldsymbol{I}$$

$$I_{2\sigma} = \frac{1}{2}[(\text{tr}\sigma)^2 - \text{tr}\sigma^2], \quad (4)$$

$$I_{3\sigma} = \det \sigma = \frac{1}{6}(\text{tr}\sigma)^3 - \frac{1}{2}\text{tr}\sigma\text{tr}\sigma^2 + \frac{1}{3}\text{tr}\sigma^3,$$

- the **cylindrical invariants** based on the roots of the characteristic equation  $\lambda_1 \geq \lambda_2 \geq \lambda_3$  and which can be expressed as follows

$$Z_\lambda = \sqrt{\frac{1}{3}}(\lambda_1 + \lambda_2 + \lambda_3),$$

$$R_\lambda = \sqrt{\frac{1}{3}(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2}, \quad (5)$$

$$\omega_\lambda = \arcsin \frac{\lambda_2 - \lambda_3}{R_\lambda \sqrt{2}}$$

- the **axi-toric-deviatoric invariants**

$$I_{1\sigma} = I \cdot \sigma, \quad I'_{2s} = \frac{1}{2}s \cdot s, \quad I'_{3s} = \frac{1}{3}(s \cdot s) \cdot s \quad (6)$$

- representation based on Lode's [18] and Novozhilov's [20, 21] approaches – instead of  $I'_{3s}$  the stress angle  $\theta$  is applied

$$\cos 3\theta = \frac{3\sqrt{3}}{2} \frac{I'_3}{(I'_2)^{3/2}} \quad (7)$$

In Eqs (1)–(7) the following notations are used:  $I$  is the second rank unit tensor,  $s = \sigma - \frac{1}{3}I \cdot \sigma I$  is the stress deviator,  $Z_\lambda, R_\lambda$  and  $\omega_\lambda$  are coordinates of a cylindrical coordinate system.<sup>3</sup>

The presented sets achieve acceptance in the theory of plasticity and in strength of materials. Some of them allows an easy split into incompressible and compressible material behavior. In addition, for some invariants a straightforward geometric interpretation can be obtained. It should be noted that the advantages of phenomeno-logical criteria formulated with the help of the introduced invariants are that they result a simple and complete description of experimental data, that they can be improved or corrected by experiences and that in the case of new materials they can be applied successfully.

#### 4. Extension to Compressible Material Behavior

In the theory of plasticity mostly incompressible material behavior is assumed and the suggested criteria of the limit state can be used. This assumption is based on experimental observations for many materials. In [22] is stated that the volumetric (hydrostatic) part of

the stress tensor is responsible for the elastic behavior only and the remaining (deviatoric) part for both the elastic and the inelastic behavior. In this case the modeling of the isotropic incompressible behavior is based on neglecting of the first invariant  $I_{1\sigma}$ , and one gets the following criteria

$$\Phi(I_{2S}, I_{3S}, \sigma_{eq}) = 0$$

or

$$\Phi(I_{2S}, \theta, \sigma_{eq}) = 0$$

From experimental observations' point of view such an idealization cannot be accepted in many cases. It is an acceptable concept especially if  $I_{1\sigma} \leq 0$ . On the other hand for some applications one needs extensions to compressible behavior. One possible extension of incompressible models are suggested in [14, 15, 23]

– by a linear transform

$$\sigma_{eq} \rightarrow \frac{\sigma_{eq} - \gamma_1 I_{1\sigma}}{1 - \gamma_1}$$

– by a quadratic transform

$$\sigma_{eq}^2 \rightarrow \frac{\sigma_{eq} - \gamma_1 I_{1\sigma}}{1 - \gamma_1} \frac{\sigma_{eq} - \gamma_2 I_{1\sigma}}{1 - \gamma_2}$$

– and a cubic transform

$$\sigma_{eq}^3 \rightarrow \frac{\sigma_{eq} - \gamma_1 I_{1\sigma}}{1 - \gamma_1} \frac{\sigma_{eq} - \gamma_2 I_{1\sigma}}{1 - \gamma_2} \frac{\sigma_{eq} - \gamma_3 I_{1\sigma}}{1 - \gamma_3}$$

Another extension to compressible behavior is introduced in [1] with the help of the basic invariants (1)

– the linear invariant

$$\sigma_1(\sigma) = \mu_1 I_{1\sigma},$$

– the quadratic invariant

$$\sigma_2^2(\sigma) = \mu_2 I_{1\sigma}^2 + \mu_3 I_{2\sigma},$$

– and the cubic invariant

$$\sigma_3^3(\sigma) = \mu_4 I_{1\sigma}^3 + \mu_5 I_{1\sigma} I_{2\sigma} + \mu_6 I_{3\sigma},$$

The  $\mu_i$ ,  $i = 1, \dots, 6$  are material parameters. The combination result in an equivalent stress as follows

$$\sigma_{eq} = \alpha \sigma_1 + \beta \sigma_2 + \gamma \sigma_3 \quad (8)$$

which is sensitive to the compressibility and several non-classical effects (see [1, 3] among others). The coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  are introduced as weight coefficients. The basic idea of representation (8) is used also in other applications, for example in [17] for creep mechanics problems. It is helpful to have the von Mises criterion as a special case that means  $\beta$  is usually equal to 1.

### 5. Characteristics of the Material

The material behavior is characterized at the limit state by one or several parameters. They should be estimated from tests. But it is well known from the material testing that only a few basic tests exist: tension test, compression test, torsion test and hydrostatic compression test. All other test are for the characterization of the multi-axiality of the material behavior. For better comparison of tests let us introduce the following parameter ratios based on [3, 4, 26]

- uniaxial compression – uniaxial tension ratio.  
Let us assume that the stress state is given by

$$\sigma = -d\sigma_+ pp = \sigma_- pp$$

and  $p$  is the unit vector characterizing the loading direction at uniaxial tension and compression. In this case one can introduce the ratio

$$d = |\sigma_-| / \sigma_+$$

$\sigma_-$  is the limit value at compression.

- torsion – uniaxial tension ratio.  
Let us assume that the stress state is given by

$$\sigma - k\sigma_+(pm + mp) = \sqrt{3}\tau_*(pm + mp)$$

and  $m$  is a orthogonal to  $p$  unit vector. The following parameter can be suggested

$$k = \sqrt{3}\tau_* / \sigma_+$$

$\tau_*$  is the limit value at torsion.

- biaxial compression – uniaxial tension ratio. Let us assume that the stress state is given by

$$\sigma = -b_D\sigma_+(pp + mm) = \sigma_{BD}(pp + mm)$$

Now one obtains the parameter

$$b_D = |\sigma_{BD}| / \sigma_+$$

$\sigma_{BD}$  is the limit value at biaxial compression.

- hydrostatic tension – uniaxial tension ratio

$$a_+^{\text{hyd}} = \sigma_+^{\text{hyd}} / \sigma_+, \quad a_-^{\text{hyd}} = |\sigma_-^{\text{hyd}}| / \sigma_+$$

Such type of parameters can be applied if the compressibility is taken into account.

- The Poisson's ratio in the inelastic case and other ratios should be presented separately.  
The plastic Poisson's ratio

$$\nu^{pl} = -\frac{\partial\Phi}{\partial\sigma_{11}} / \frac{\partial\Phi}{\partial\sigma}$$

follows from the normality rule with  $\sigma_{II} = \sigma_{III} = 0$ . One distinguishes the Poisson's ratio at tension  $\nu_+^{pl}$  with  $\sigma_1 = \sigma_{eq}$  and at compression  $\nu_-^{pl}$  with  $\sigma_1 = -d\sigma_{eq}$ . The parameter allows to restrict the model parameters.

By analogy to the Poisson's ratio the elongation – contraction ratio for a torsion bar can be established

$$\chi = \frac{\partial\Phi}{\partial\sigma_{11}} / \frac{\partial\Phi}{\partial\sigma_{12}}$$

with  $\sigma_{12} = k\sigma_{eq}/\sqrt{3}$  and  $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{13} = \sigma_{23} = 0$ . With the help of this ratio similar to the Poynting-effect, the Poynting-Swift-effect and the Kelvin-effect [3, 8] effects can be characterized by further properties.

For  $\Phi(I_{2s}, I_{3s}, \sigma_{eq})$  (incompressible case) the range of  $\chi$  is  $\chi \in \left[-\frac{1}{6}; \frac{1}{6}\right]$ . The calculations in

the case of compressible material behavior the range follows from very complex calculations.

Let us assume again the special cases of Vonmises-type material behavior. In this case one gets the material behavior ratios presented in Table 1 for two variants of criteria. Note that the ratios  $d$  and  $k$  are applied in [1]. They coincide in both approaches.

Table 1

Material behavior ratios for two limit criteria	
VON MISES	Normal Stress Hypothesis
$d = 1$	$D \rightarrow \infty$
$K = 1$	$k = \sqrt{3}$
$b_D = 1$	$b_D \rightarrow \infty$
$a_+^{hyd}, a_-^{hyd} \rightarrow \infty$	$a_+^{hyd} = 1, a_-^{hyd} \rightarrow \infty$
$\nu_-^{pl} = \nu_+^{pl} = 1/2$	$\nu_-^{pl} \rightarrow \infty, \nu_+^{pl} = 0$
$\chi = 0$	$\chi = 0$

## 6. Requirements to Strength Criteria

The correctness of any hypothesis can be verified by experimental data. On the other hand, there are not enough accurate data at combined stress states. For example, the scattering of the data allows a satisfying fit of different models by the same experimental data set. From this it follows that the uniqueness of the choice of a criterion cannot be established: there are no sufficient conditions for the choice.

Let us introduce at first some *necessary conditions* presented in the literature:

- convexity  
This requirement is not necessary for all failure criteria.
- trigonal or hexagonal symmetry of the limit surface in the  $\pi$ -plane  
The rotational symmetry can be obtained for models with smooth surfaces as an interim solution;
- bounds of plastic Poisson's ratio at tension  $\nu_+^{pl} \in ]-1, 1/2]$ ;
- bounds of plastic Poisson's ratio at compression  $\nu_+^{pl} \in ]-1, 1/2]$  for materials with restricted hydrostatic compression, otherwise if  $a_+^{hyd} \rightarrow \infty$  it follows  $\nu^{pl} \geq 1/2$ ;
- bounds of the hydrostatic tension  $a_+^{hyd} \in ]1/3; 1/(1-2\nu_+^{pl})]$

This value can be bounded additionally

$$a_+^{hyd} = \left[ \frac{1}{\sqrt[12]{3^{11}(1-2\nu_+^{pl})}}, \frac{1}{1-2\nu_+^{pl}} \right]$$

The upper bound follow from the Drucker-Prager criterion, the lower bound from rotational symmetric ellipsoid with the power  $n = 12$

$$\frac{(3I_{2s})^6 + a_{12}I_{\sigma}^{12}}{1 + a_{12}} = \sigma_{eq}^{12}, \quad a_{12} \geq 0$$

Assuming the lowest bound  $a_+^{hyd} = 1/3$  yields from the plane  $I_{2\sigma} = \sigma_{eq}$

$$I_{1\sigma} = \sigma_{eq}$$

Since one has not enough necessary and sufficient conditions one introduces *plausibility conditions* following from engineering understanding: the adequate description of the experimental data, the reliability and trustworthiness of the predictions, the simple and confident application, the understandability of the models, a clear geometrical background, a physical basis (not only abstract mathematical formulation), the account of the medial stress, a minimal number of parameters, dimensionless parameters, continuous differentiability even for the limit surfaces, continuous differentiability in the hydrostatic nodes ("rounded apex" after Franklin), and the models should contain well-known hypotheses as special cases. *Additional requirements* are: the explicit resolution with respect of  $\sigma_{eq}$ , the wide as possible range of convex shapes ( $\pi$ -plane)  $\rightarrow$  singular edges, no change of the shape ( $\pi$ -plane) for intersections  $I_{1\sigma} = \text{const.}$ , no formulation of model surfaces with partial surfaces, which result in singular meridians (only models with planes or smooth surfaces can be used), the dependence of the models for  $\Phi$  of all three invariants, only rational functions of the invariants  $I_{1\sigma}$  and  $I_{3s}$  should be introduced, and the maximum of the stress power cannot be higher than 12.

## 7. Conclusions

Considering the big number of models suggested, up to now there are no physical statements for the shape of the surface  $\Phi$ . From this it follows that the models can be only formulated empirically. There are many proposals considering the convexity of the limit surface. It can be shown that range of convex incompressible models is bounded. In [15] the incompressible models are presented in the  $k-d$  diagram. Assuming that three parameters from the tension, compression and torsion tests are enough to represent the convex forms of incompressible material behavior it can be shown the bounds for such models. The convexity follows from Drucker's postulate [11]. But there are a lot of experimental observations demonstrating non-convex forms of material behavior.

## 8. Outlook

The formulation and investigation of limit criteria will be in the focus of the scientific community in the future. The reason is that one has new materials and application fields. The limit surface approach is not related to the microstructure, but it is a simple engineering way to solve problems related to the strength prediction or material behavior modeling. It will be discussed in the nearest future:

- how one can get a better experimental data set for making an adequate model choice;
- are there any formulation principles based on the mathematical structure of the criteria or symmetry considerations allowing the enough accurate generalized criterion, and last but not least;
- can be the microstructure taken into account.

Since one has a partly significant scattering of the experimental data rules for the material parameters estimates must be established. One possible approach are convexity studies for the limit surface. The investigation of the convexity bounds for simple criteria with one or two parameters can be performed as it is shown in [5, 24]. It can be shown that for advanced criteria only numerical investigations result in knowledge-based bounds [9].

The mathematical structure of generalized criteria is discussed, for example, in [1, 13]. One possible generalized criterion is given by Eq. (8), another possibility is given as a quadratic equation

$$\left(\sigma_{\text{eq}}^n\right)^2 + S_n \sigma_{\text{eq}}^n + S_{2n} = 0, \quad n \in |\mathbb{N}|$$

or a cubic one

$$\left(\sigma_{\text{eq}}^m\right)^3 + S_m \left(\sigma_{\text{eq}}^m\right)^2 + S_{2m} \sigma_{\text{eq}}^m + S_{3m} = 0, \quad m \in |\mathbb{N}|$$

The integrity base  $S, i \in |\mathbb{G}|$  is a set of scalar functions of the axiatoric-deviatoric invariants (6) [7]

$$S_1 = a_1 I_{1\sigma},$$

$$S_2 = a_2 I_{1\sigma}^2 + b_2 I_{2s},$$

$$S_3 = a_3 I_{1\sigma}^3 + c_3 I_{3s} + d_2 I_{1\sigma} I_{2s},$$

...

$$S_6 = a_6 I_{1\sigma}^6 + b_6 I_{2s}^3 + c_6 I_{3s} + d_6 I_{1\sigma}^2 I_{2s}^2 + e_6 I_{1\sigma}^4 I_{2s} + f_6 I_{1\sigma}^3 I_{3s} + g_6 I_{1\sigma} I_{2s} I_{3s}$$

...

with the parameters  $a_p, b_p, c_p, \dots$ . It can be shown that this representation contains a lot of classical and non-classical limit criteria for incompressible and compressible materials [13]. Both the quadratic and the cubic equation can be solved analytically with respect to  $\sigma_{eq}$ . In addition, each term of the integrity base is differentiable with respect to the stress tensor  $\sigma$ . But considering the great number of parameters the question of applicability arises.

With respect to new material models based on the Cosserat continuum [2] further developments should be directed to the question of suitable equivalent stress hypotheses in this case.

*The paper is partly based on the discussions with my colleague Dr.-Ing. Vladimir Kolupaev (German Institute of Plastics, Darmstadt).*

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