CZASOPISMO TECHNICZNE M
WYDAWNICTWO POLITECHNIKI KRAKOWSKIEJ

z. 8-M/2008
ISSN 0011-4561
ISSN 1897-6328

WŁADYSŁAW MITIANIEC, KONRAD BUCZEK*

TORSIONAL VIBRATION ANALYSIS OF CRANKSHAFT IN HEAVY DUTY SIX CYLINDER INLINE ENGINE

ANALIZA DRGAŃ SKRĘTNYCH WAŁU KORBOWEGO WYSOKOOBCIĄŻONEGO 6-CYLINDROWEGO SILNIKA RZĘDOWEGO

A b s t r a c t

The paper contains torsional vibration analysis of crankshaft in six cylinder inline heavy duty engine taking advantage of the crank train reduction to multi-mass model. Obtained results were compared with FEM analysis performed in ANSYS to determine the influence of the reduction a physical continuum to multi-mass model. In order to achieve the course of torsional moment in the highly load cross section of the crankshaft, there was a necessity to use Fourier analysis of tangential force acting on crank and execute the analysis of most beneficial firing order.

Keywords: *combustion engines*, *crank drive*

S t r e s z c z e n i e

Artykuł zawiera analizę drgań skrętnych sześciocylindrowego rzędowego silnika trakcyjnego, przeprowadzoną za pomocą redukcji układu korbowego do wielomasowego modelu. Otrzymane wyniki porównano z wynikami analizy MES wykonanej w programie ANSYS, umożliwiając określenie wpływu redukcji układu ciągłego do modelu wielomasowego. W celu otrzymania przebiegu momentu skręcającego w najsilniej obciążonym przekroju poprzecznym wału konieczne było przeprowadzenie analizy Fouriera siły stycznej działającej na wykorbienie wału oraz analizy najkorzystniejszej kolejności zapłonów.

Słowa kluczowe: *silniki spalinowe*, *układ korbowy*

* Ph.D. D.Sc. Eng. Władysław Mitianiec, M.Sc. Eng. Konrad Buczek, Institute of Automobiles and Internal Combustion Engines, Cracow University of Technology.

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

1. Introduction

Every material system containing individual mass and stiffness distribution is susceptible to vibrate. These vibrations can be caused either by single impulse of load or a periodical load. In the first case a free vibration occurs and in the second case a forced vibration is achieved. A free vibration is not significance in technical applications because there is always a periodical load, which cause a forced vibration. Nevertheless, analysis of free vibration is essential to determine the natural frequencies of material system, which is responsible of resonance phenomenon. Resonance occurs during equalizing the periodical load frequency with one of the natural frequencies and it leads to high amplitude of vibration and high fatigue stresses into the material.

Torsional vibration is one of the most important problems in crankshaft design of big bore engines (D > 90 mm). Relatively big mass of piston assembly and connecting rod increases resultant moment of inertia of every crank mechanism thus decreases the natural frequency of first and second mode to the range of engine rotational speed. It results in resonance with low order harmonic of tangential force acting on crankpin, which is very undesirable in connection with occurring of significant additional torsional moment.

Increasing value of natural frequency can be obtain by applying material with bigger Young module (for example steel instead of cast iron), decreasing resultant moment of inertia of crank mechanisms (lighter piston assembly and connecting rod, hollowed crank pins or whole crankshafts) or increasing crankshaft stiffness (by increasing main journal and crank pin diameter). There are some disadvantages of these modifications. Using steel as a crankshaft material gives more restrictions in possible shape of the crankshaft than applying cast iron, reducing mass of pistons or connecting rods is usually not able to perform because they are often highly optimized, hollow crank pins force additional machining and increasing manufacturing cost. Even increasing crank pin and main journal diameter increases mass of crankshaft thus make crankshaft production more expensive.

According to these considerations, we can easily notice that optimization of crankshaft design is an iterative process leading to satisfactory result, when finding the most optimal solution is hardly possible.

2. Multi-mass model analysis

2.1. Reduction to multi-mass model and determining natural frequencies of torsional vibration

Reduction of crank train to multi-mass model strongly simplified the torsional vibration calculations, nevertheless giving results, which correctly reflects the reality. The model for six cylinder inline engine is shown in Fig. 1.

The foregoing model consists of elements with specified moments of inertia θ_i connected between themselves by elements with specified torsional flexibilities $e_{i\text{-}j}$. The specific parameters describes:

θ_1 – reduced to crankshaft axis moment of inertia of auxiliary drives connected with crankshaft front end,

θ_2 – moment of inertia of crankshaft front end with its tooth wheel,

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

θ_3–θ_8 – moment of inertia of crankshaft part and reduced to crankshaft axis moments of inertia of connecting rod and piston assembly,
θ_9 – moment of inertia of crankshaft rear end and flywheel,
e_{1-2} – torsional flexibility of auxiliary drives,
e_{2-3} – torsional flexibility of crankshaft front end and first crank web,
e_{3-4}–e_{7-8} – torsional flexibility crankshaft crank webs,
e_{8-9} – torsional flexibility of last crank web and rear end of crankshaft.

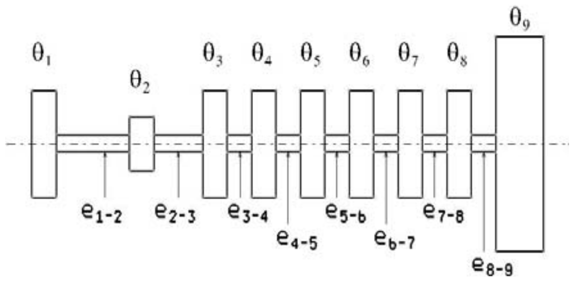

Fig. 1. Multi-mass model of six cylinder inline engine for torsional vibration calculations

Rys. 1. Model wielomasowy 6-cylindrowego silnika rzędowego dla obliczeń drgań skrętnych

Analysis of torsional vibration taking advantage of multi-mass model can be split into following steps:
– determination of model parameters,
– natural frequencies calculation connected with determination of vibration modes,
– firing order analysis,
– resonant torsional moment analysis.

Fundamental equations circumscribing the behavior of multi mass model can be expressed using Lagrange formula

$$\frac{d}{d\tau}\left(\frac{\partial E}{\partial \dot{B}_i}\right)+\frac{\partial V}{\partial B_i}=0 \tag{1}$$

where:
E – kinetic energy of whole model,
V – potential energy of whole model,
B_i – instantaneous angle deflection of element i,
τ – time.

Executing differentiation leads to system of equations

$$\theta_i\ddot{B}_i+\frac{1}{e_{i-1,i}}(B_i-B_{i-1})+\frac{1}{e_{i,i+1}}(B_i-B_{i+1})=0\,; \quad i=1,2,...,n \tag{2}$$

Assuming solution as follows

$$B_i = A_i\cos(\Omega\tau) \tag{3}$$

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

the system of equation can be easily expressed using matrix algebra

$$\left([\theta]\Omega^2 - [K]\right)[A] = 0 \tag{4}$$

where

$$[\theta] = \begin{bmatrix} \theta_1 & 0 & 0 & \ldots & 0 \\ 0 & \theta_2 & 0 & \ldots & 0 \\ 0 & 0 & \theta_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & \theta_n \end{bmatrix} \tag{5}$$

$$[K] = \begin{bmatrix} \frac{1}{e_{1,2}} & \frac{-1}{e_{1,2}} & 0 & 0 & \ldots & 0 & 0 & 0 \\ \frac{-1}{e_{1,2}} & \frac{1}{e_{1,2}} + \frac{1}{e_{2,3}} & \frac{-1}{e_{2,3}} & 0 & \ldots & 0 & 0 & 0 \\ 0 & \frac{-1}{e_{2,3}} & \frac{1}{e_{2,3}} + \frac{1}{e_{3,4}} & \frac{-1}{e_{3,4}} & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & \ldots & \frac{-1}{e_{n-2,n-1}} & \frac{1}{e_{n-2,n-1}} + \frac{1}{e_{n-1,n}} & \frac{-1}{e_{n-1,n}} \\ 0 & 0 & 0 & 0 & \ldots & 0 & \frac{-1}{e_{n-1,n}} & \frac{1}{e_{n-1,n}} \end{bmatrix} \tag{6}$$

$$[A] = \begin{bmatrix} A_1 \\ A_2 \\ \ldots \\ A_n \end{bmatrix} \tag{7}$$

Multiplying equation (4) by converse of matrix [θ] we achieve formula

$$\left|[I]\Omega^2 - [B]\right| = 0 \tag{8}$$

which presents secular equation of matrix [B]. This matrix has no physical interpretation, nevertheless eigenvalues of matrix [B] presence the square of natural frequencies of multi-mass model. Additionally eigenvectors of foregoing matrix describes different modes of torsional vibration. Solution of equation (8) is currently very easy using one of many commercial mathematical PC softwares.

2.2. Work of tangential forces

The above mentioned calculations gives information about engine rotational speed which resonate with specific harmonic of tangential force acting on crankpin, but do not specify the torsional torque caused by vibration. To find it, there is a necessity to specify

BPK BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

work of all tangential forces acting on crankshaft and work of all damping forces. Equality of this works, gives possibility to calculate one of amplitude, which was used as parameter in both work equations.

Infinitesimal work of force acting on crankpin can be expressed as

$$\mathrm{d}L = M\,\mathrm{d}B \tag{9}$$

where:

M – torque of specific harmonic caused by tangential forces acting on one crank mechanism,

dB – infinitesimal torsion.

The harmonic M and torsion B can be assumed as

$$M = [M]\sin(\chi) \tag{10}$$

$$B = A\sin(\chi - \beta) \tag{11}$$

where:

$[M]$ – amplitude of torque of considered harmonic of tangential forces,

A – torsional vibration amplitude of considered crank,

χ – angular variable,

β – angular delay of torsion relating to torque harmonic.

Substituting (10) and (11) to (9) one achieves

$$dL = [M]\sin(\chi)d[A\sin(\chi - \beta)] \tag{12}$$

and after integration

$$L = [M]A\int_0^{2\pi}\sin(\chi)d\sin(\chi - \beta) = \pi[M]A\sin\beta \tag{13}$$

Expression (13) takes into consideration only one crank mechanism. Including all crank train we achieved

$$L = \pi\sum_{i=1}^{c}[M]_i A_i \sin\beta_i \tag{14}$$

There is necessity to remind, that all amplitude A_i are in the same phase during vibration, so all β_i are caused by angles between crank webs, order of torque harmonic of tangential force and firing order.

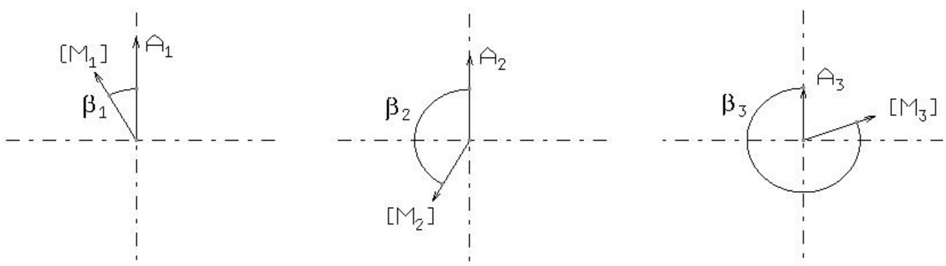

Fig. 2. Relative position of vectors A and M

Rys. 2. Względne położenie wektorów A i M

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

The consideration about work of tangential forces acting on crankshaft can be simplified if we perform a mathematical transformation, which based on assumption, that all vectors M are in the same phase and then (14) can be expressed as

$$L = \pi[M]\sum_{i=1}^{c} A_i \sin\beta_i \tag{15}$$

The situation after transformation is visualize on Fig. 3.

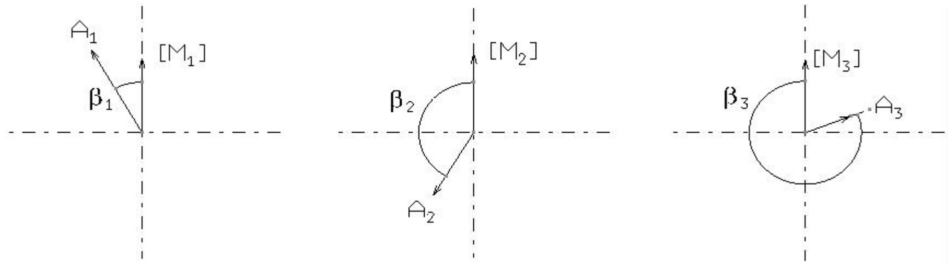

Fig. 3. Relative position of vectors A and M after transformation

Rys. 3. Względne położenie wektorów A i M po przekształceniach

Additionally using relative amplitude defined as

$$A_i = A_1 \alpha_i \tag{16}$$

and received as eigenvectors from equation (8), we obtain

$$L = \pi[M]A_1\sum_{i=1}^{c} \alpha_i \sin\beta_i \tag{17}$$

The summation into (17) can be assumed as

$$\sum_{i=1}^{c} \alpha_i \sin\beta_i = \sigma \sin\beta \tag{18}$$

which is definition of resultant relative amplitude σ. The work can be finally expressed as

$$L = \pi[M]A_1\sigma\sin\beta \tag{19}$$

During resonance, foregoing work becomes maximal, so

$$L_{res} = \pi[M]A_1\sigma \tag{20}$$

The only unknown in (20) is amplitude A_1 which is usually an amplitude of free end of the crankshaft during vibration. Equality of work of tangential forces acting on crankshaft and work of friction during resonance allows to calculate value of amplitude A_1 and hence also other amplitudes A_i.

BPK BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

2.3. Work of damping

The damping occurred during torsional vibration is mainly caused by piston friction. Other sources of damping are internal friction of crankshaft material and viscose friction into crankshaft bearings, but they influence are negligible, especially when steel crankshafts is considered.

Reminding that piston friction is semi-fluid friction, the formula of friction moment can be expressed as following

$$M_t = \mu \frac{dB}{d\tau} \tag{21}$$

where:

M_t – moment of torsional vibration damping,
μ – damping coefficient,
B – torsion.

Assuming that torsion can be determined as

$$B = A\sin(\Omega\tau) \tag{22}$$

Infinitesimal work of damping can be expressed as follows

$$\mathrm{d}L_t = M_t \mathrm{d}B = \mu A\Omega\cos(\Omega\tau)\mathrm{d}[A\sin(\Omega\tau)] \tag{23}$$

After substituting

$$\Phi = \Omega\tau \tag{24}$$

we obtain

$$L_t = \mu\Omega A^2 \int_0^{2\pi} \cos^2(\Phi)d\Phi \tag{25}$$

and after integration

$$L_t = \pi\mu\Omega A^2 \tag{26}$$

Including all crank mechanisms the work can be expressed as

$$L_{tc} = \pi\mu\Omega\sum_{i=1}^{c} A_i^2 \tag{27}$$

Taking advantage of (16), the summation in (27) can be written down using relative amplitude

$$L_{tc} = \pi\mu\Omega A_1^2 \sum_{i=1}^{c} \alpha_i^2 = \pi\mu\Omega A_1^2 (\alpha^2)_w \tag{28}$$

An essential problem presents a determination of damping coefficient μ, which assumes wide range of values in literature. The most common is Holtzer expression

$$\mu = 0{,}04\theta\Omega \tag{29}$$

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

where the damping coefficient is treated as a function of:

θ – moment of inertia of one crank mechanism (including influence of piston and connecting rod),

Ω – frequency of crankshaft rotation.

Finally the work of damping using (29) can be expressed as

$$L_{tc} = \pi 0{,}04\theta\Omega^2 A_1^{\,2} (\alpha^2)_w \tag{30}$$

2.4. Determination of amplitudes during resonance

During resonance, the work of forces acting on crankshaft and work of damping are equal. The equality of (20) and (30) gives possibility to calculate only one unknown A_1 present in both equations as

$$A_1 = \frac{M_{(h)}\sigma_{w(h)}}{0{,}04\theta\Omega_w^{\,2}(\alpha^2)_w} \tag{31}$$

Additional indexes in (31) means:

h – order of considered harmonic,

w – mode of torsional vibration resonate with harmonic *h*.

All other amplitudes can be obtained according to (16). In order to minimize the torsional torque caused by resonance, the different firing orders have to be considered. The firing order determines the value of resultant relative amplitude $\sigma_{w(h)}$ by varying β_i angles in summation (18). The analysis has to be carrying out for different harmonics and modes to obtain the most satisfactory results.

3. Results of multi-mass model analysis

The analysis has been performed on crankshaft for six cylinder heavy duty inline engine (11dm^3 engine displacement) during design phase (Fig. 4). The counterweights balanced 50% of rotating masses of connecting rods.

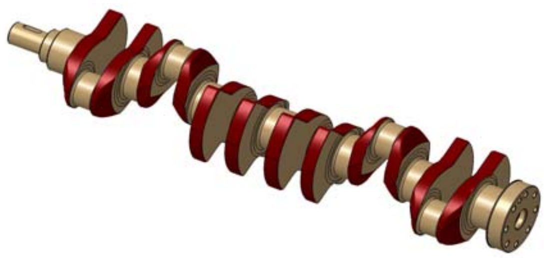

Fig. 4. Considered model of crankshaft

Rys. 4. Model wału korbowego

The specific moments of inertia of crankshaft parts (Fig. 1) have been determined using CAD system. The other moments of inertia (auxiliary drives, flywheel etc.) have also been included. Results of foregoing determination are showed in Table 1.

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

Table 1

Results of mass reduction

Quantity	Design.	Value	Unit
Moment of inertia of auxiliary drive reduced to crankshaft axis	θ_1	0,063	kg m^2
Moment of inertia of front end of crankshaft with it's tooth wheel	θ_2	0,027	kg m^2
Resultant moment of inertia of 1st cylinder crank mechanism	θ_3	0,190	kg m^2
Resultant moment of inertia of 2nd cylinder crank mechanism	θ_4	0,109	kg m^2
Resultant moment of inertia of 3rd cylinder crank mechanism	θ_5	0,190	kg m^2
Resultant moment of inertia of 4th cylinder crank mechanism	θ_6	0,190	kg m^2
Resultant moment of inertia of 5th cylinder crank mechanism	θ_7	0,109	kg m^2
Resultant moment of inertia of 6th cylinder crank mechanism	θ_8	0,190	kg m^2
Moment of inertia of rear end of crankshaft including flywheel	θ_9	5,82	kg m^2

The flexibility of crankshaft parts has been obtained by FEM calculations. Every part of crankshaft has been turned by tentative torque, what allows to measure a torsion (Fig. 5). These values provide enough data to calculate flexibility $e_{1\text{-}2}$–$e_{8\text{-}9}$ (Table 2). Calculation of multi-mass model has been performed in MathCAD system. Obtained results are shown in Table 3 and Fig. 6.

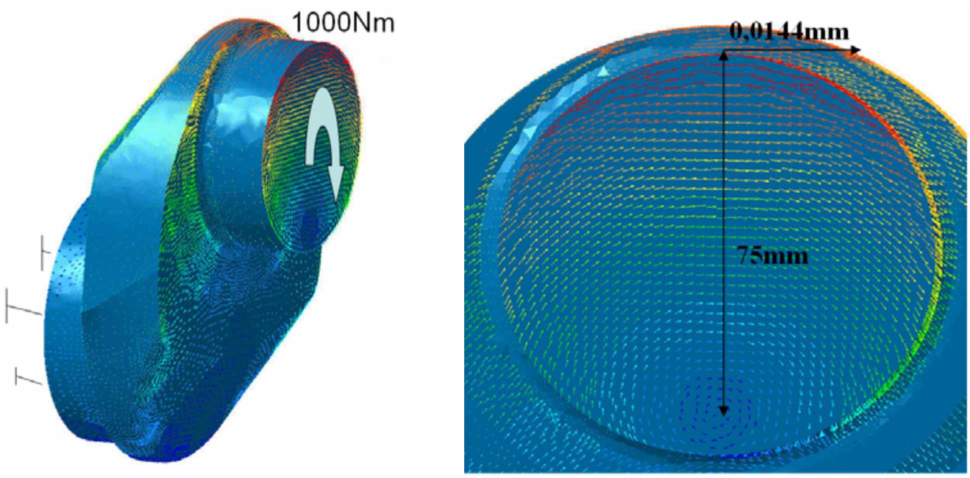

Fig. 5. Determining flexibility by FEM calculations

Rys. 5. Wyznaczenie elastyczności za pomocą obliczeń MES

BPK BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

Table 2

Results of flexibility reduction

Quantity	Designation	Value	Unit
Flexibility of auxiliary drive connected to front end of crankshaft	e_{1-2}	2,850E-06	rad/Nm
Flexibility of element connecting front end tooth wheel with 1st cylinder reduced mass	e_{2-3}	1,154E-06	rad/Nm
Flexibility of element connecting 1st and 2nd cyl. reduced masses	e_{3-4}	3,840E-07	rad/Nm
Flexibility of element connecting 2nd and 3rd cyl. reduced masses	e_{4-5}	3,840E-07	rad/Nm
Flexibility of element connecting 3rd and 4th cyl. reduced masses	e_{5-6}	4,080E-07	rad/Nm
Flexibility of element connecting 4th and 5th cyl. reduced masses	e_{6-7}	3,840E-07	rad/Nm
Flexibility of element connecting 5th and 6th cyl. reduced masses	e_{7-8}	3,840E-07	rad/Nm
Flexibility of element connecting 6th cyl. reduced mass with flywheel	e_{8-9}	5,590E-08	rad/Nm

Table 3

Natural frequencies of multi-mass model analysis

Mode	Natural frequencies (multi-mass model results)	Natural frequencies (FEM calculation results)	Relative error
	n_{mm}	n_{FEM}	Δn
	[1/min]	[1/min]	%
1	9954	10623	6,3
2	19726	19953	1,1
3	31177	–	–
4	49757	–	–
5	66907	–	–
6	70789	–	–
7	77510	–	–
8	102649	–	–

BPK BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

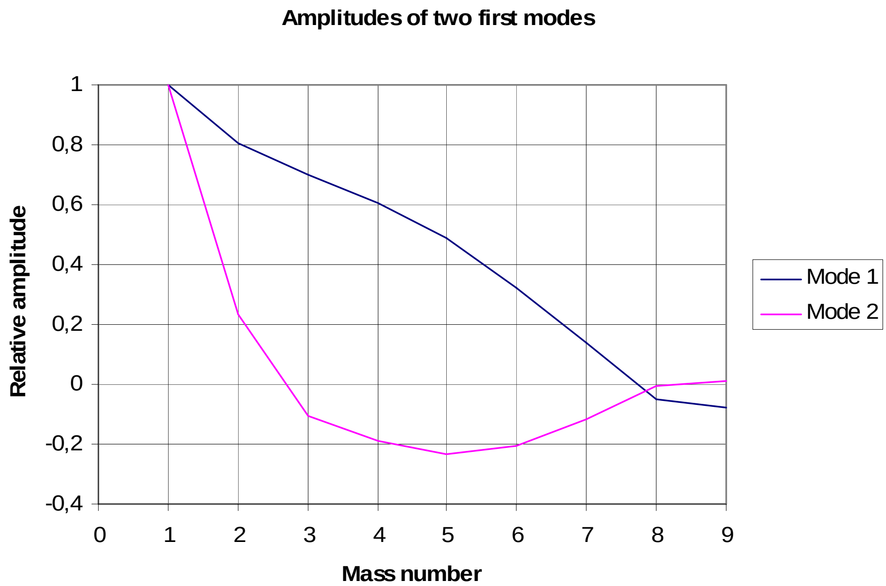

Fig. 6. Relative amplitudes of first two modes

Rys. 6. Amplitudy względne pierwszych dwóch postaci drgań

4. Results of FEM calculations

In order to compare natural frequencies obtained by two different methods, the FEM calculation has been performed. The geometry of crankshaft has been exported to ANSYS software. Simplified masses of auxiliary drive, flywheel, connecting rods and pistons have been added to include the influence of these elements on natural frequencies. Masses of pistons and connecting rods have been represented by circular rings attached to crankpins, which mass contains rotating mass of connecting rod and half of oscillating mass (mass of piston and oscillating mass of connecting rod) – Fig. 7. The picture shows the surfaces of frictionless support. Not all of main bearings have been supported in order not to stiffen the

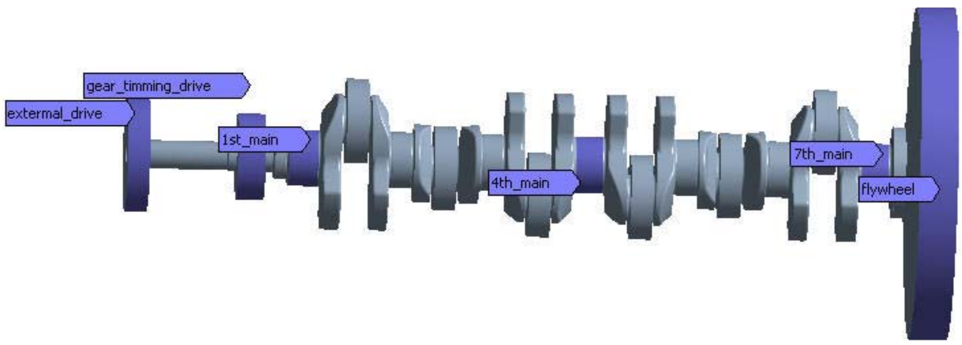

Fig. 7. 3D model exported to ANSYS software

Rys. 7. Model 3-wymiarowy eksportowany do programu ANSYS

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

model, which would lead to increasing natural frequencies results. The FEM results of natural frequencies are shown in Table 3. The relative error of natural frequencies obtained from FEM and multi-mass model calculations for first two modes hasn't been significant. This proves, that the multi-mass model analysis highly reflects the reality.

5. Firing order analysis

Proper firing order can significantly decrease the torque caused by torsional vibration in crankshaft. In order to define foregoing torque, there is necessity to determine work of tangential forces and work of damping forces.

Fourier analysis of tangential force (*T*) is shown in Fig. 8. Harmonics with the biggest amplitude are most dangerous (h = 0,5, 1, 1,5, 2,5, 3,5), but they don't resonate even with first mode of torsional vibration because of low max. engine speed (1900 rpm). The resonant engine speeds are shown in Fig. 9. There is possibility to resonance with h = 5,5 and higher for first mode of torsional vibration.

Analyzing relative resultant amplitude (Fig. 10), the firing order 1-5-3-6-2-4 has been chosen, as the most proper. For this firing order, the resultant relative amplitude, which determines the work of tangential forces (20), is much smaller during resonance with harmonic h = 5,5 than for firing order 1-2-4-6-5-3.

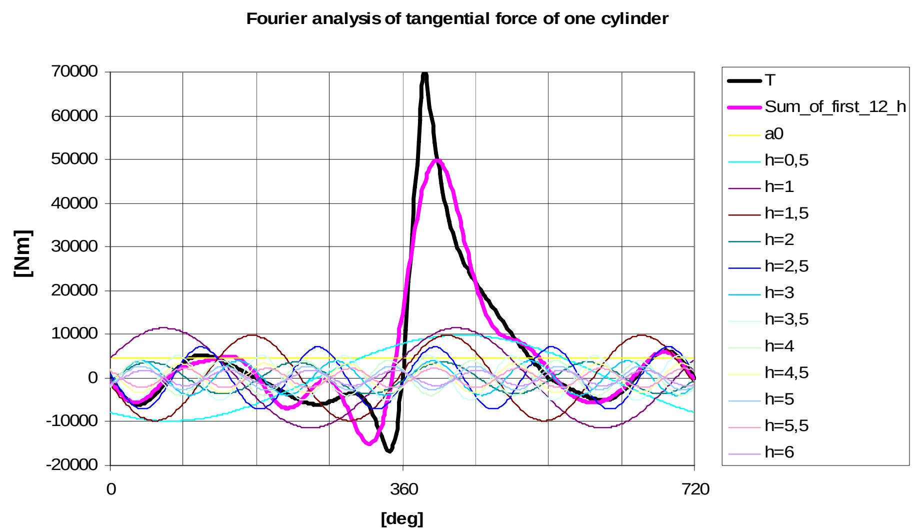

Fig. 8. Fourier analysis of tangential force

Rys. 8. Analiza Fouriera sił stycznych

BPK BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

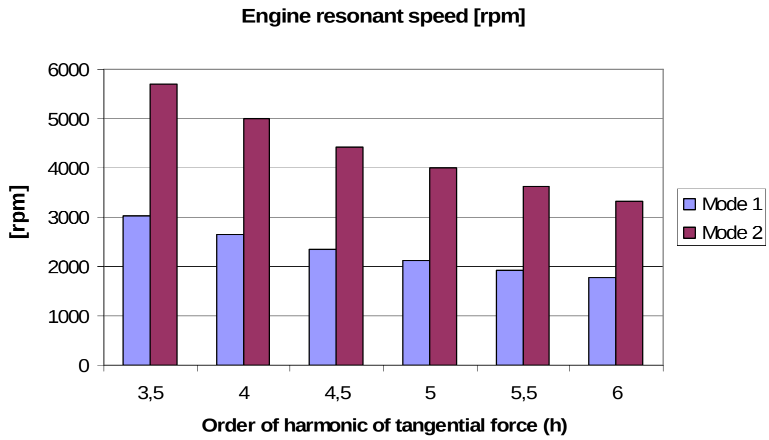

Fig. 9. Engine resonant speed

Rys. 9. Prędkości rezonansowe silnika

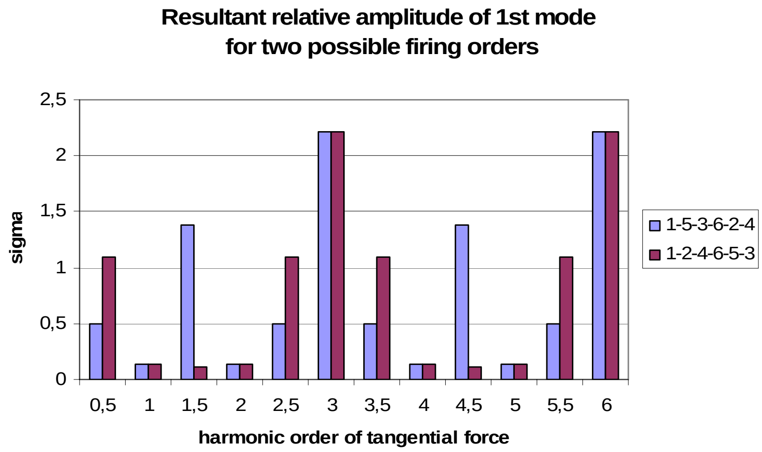

Fig. 10. Relative resultant amplitude of 1st mode

Rys. 10. Względne amplitudy wypadkowe I formy drgań

The most dangerous resonance occurs with harmonic $h = 6$. It is caused by high re-sultant relative amplitude for the harmonic of this order. Taking into consideration the Holtzer expression of damping work, the analysis of crankshaft torsion in resonance with foregoing harmonic has been performed. The obtained results are shown in Fig. 11. The biggest torsions caused by resonance have occurred after 5^{th} cylinder (M(6)+). It has been more than three times bigger than torque caused by tangential forces (M1-5). The sum of these torques (Mc+) shows the small influence of tangential forces on possible fatigue failure.

BPK BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

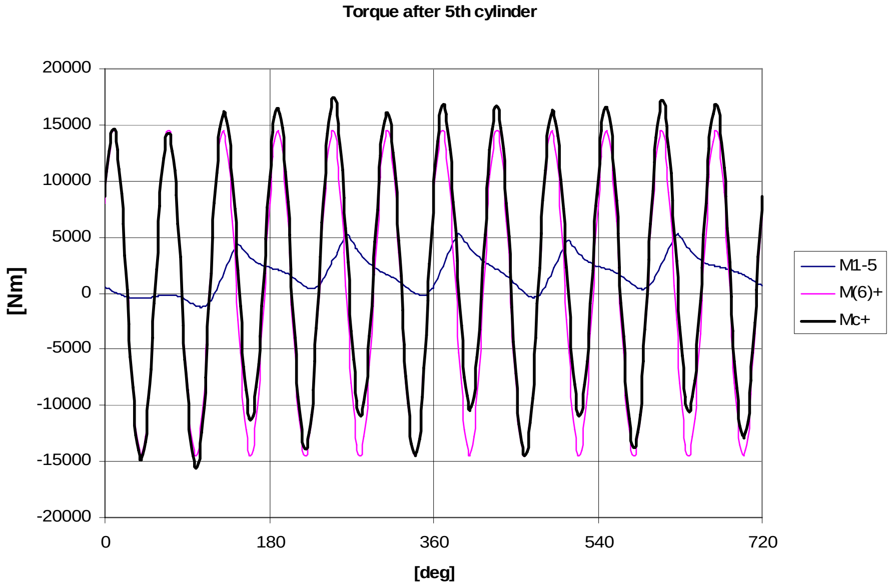

Fig. 11. Torque after 5th cylinder

Rys. 11. Moment na wale po 5 cylindrze

Such a high value of torque amplitude is not acceptable. When mass reduction is not possible, the only way to reduce effect of a resonance is to apply the torsional vibration damper (TVD). The principal of operation of TVD is increasing the work of friction (damping work) and thus decreasing the crankshaft torsion.

6. Conclusions

1. The torsional vibration presence significant problem in big bore inline engines ($D > 90$ mm) – necessity to apply torsional vibration damper.
2. Natural frequencies of crankshaft can be determined with high accuracy taking advantage of multi-mass model analysis.
3. Determining flexibilities of different parts of crankshaft for multi-mass model by FEM analysis is recommended.
4. The damping of torsional vibration occurs mainly by piston friction.
5. Resonance of tangential force harmonics is most dangerous for first mode of torsional vibration.
6. The torque caused by resonance is much bigger than torque caused by tangential forces in multi-cylinder big bore engines.

W tekstach obcojęzycznych Redakcja dokonała tylko standardowej adiustacji, zachowując ich oryginalną wersję.

BPK BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ

References

[1] Jędrzejowski J., *Mechanika układów korbowych silników samochodowych*, WKiŁ, Warszawa 1986.

[2] Wajand J.A., *Tłokowe silniki spalinowe średnio i szybkoobrotowe*, WNT, Warszawa 2005.

[3] Matzke W., *Projektowanie mechanizmów korbowych silników szybkobieżnych*, WKiŁ, Warszawa 1974.

[4] Orłow P.L., *Zasady konstruowania w budowie maszyn*.

[5] Basshuysen R., *Internal Combustion Engine Handbook*, SAE International, 2004.

[6] *Mały poradnik mechanika*, t.1 i 2, WNT, Warszawa 1994.

[7] ANSYS User Manual.

BIBLIOTEKA CYFROWA POLITECHNIKI KRAKOWSKIEJ