IMPLEMENTACJA PERIODYCZNEGO MODELU TRANSMISJI CIEPŁA W PROGRAMIE SYMULACJI CZTEROWYMIAROWEJ

THE IMPLEMENTATION OF A PERIODIC THERMAL CONDUCTION MODEL IN A 4D SIMULATION PROGRAM

Abstract
The periodic calculation approach allows us to capture heat conduction and heat storage effects in three-dimensional models. It is necessary to use this calculation approach if there is a considerable influence of heat storage capacity while calculating thermal bridges. Three-dimensional, periodically working thermal-bridge-analysis software as well as the possibilities of visualizing the results dependent on four dimensions will be presented. Their practicability is exemplified by an earth-coupling system.

Keywords: thermal bridges, heat storage capacity, 3D calculation, periodic calculation

Streszczenie
Obliczenia periodyczne pozwalają zarejestrować transmisję ciepła oraz efekty magazynowania ciepła w modelach trójwymiarowych. Zastosowanie tej metody obliczeniowej staje się konieczne pod znacznym wpływem pojemności magazynowania ciepła w trakcie kalkulacji mostków cieplnych. W niniejszym artykule zaprezentowano trójwymiarowe periodyczne oprogramowanie do analizy mostków cieplnych, jak również możliwości wizualizacji wyników zależnych od czterech wymiarów. Możliwości wprowadzania ich w życie ukazano na przykładzie systemu ziemnego.

Słowa kluczowe: mostki cieplne, pojemność magazynowania ciepła, obliczenia trójwymiarowe, obliczenia periodyczne

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1. Introduction

In the course of thermal bridge calculations, the capture of a multidimensional heat conduction process is generally based on steady-state, i.e. time-independent borderline conditions. In this calculation approach, the heat storage capacity of analyzed constructions has no influence on the calculation results.

Therefore, the steady-state calculation will only provide useful approximation results when the heat storage capacity of an investigated building is to be neglected. Old and historical buildings, with massive masonry and thick walls, will not comply with these requirements. The heat storage capacity of earth-coupled systems is of utmost importance. In these cases, the calculation has to be transient, i.e. time-dependent.

The theoretical basis for a multidimensional, transient calculation of heat conduction and heat storage operations – for the special case of time-dependent periodic borderline conditions – has been known for some time [1, 2]. The main results of this theory will be briefly recapitulated here.

During the implementation of the periodic theory into the thermal bridge calculation software AnTherm [3, 4], the presentation of calculation results proved a major challenge. The reason is time as an additional fourth dimension to the three spatial coordinates in comparison to steady-state calculations. In this paper, solutions for the visualization of two- and three-dimensional transient calculations are shown, discussed and complemented with examples.

2. The periodic calculation approach

It has been known for some time that the heat conduction equation can be solved for each harmonic in the special case of periodic set borderline conditions. This means that there is no need for time discretization [1]. The further problem of unknown starting conditions does not arise.

The theory of thermal conductances was developed for the steady-state case and presented in the book “Wärmebrücken” (Thermal Bridges) [5]. It proved to be a special case of the more common periodic theory [1]. The fundamental relations are the same for the steady-state case as well as for the periodic theory. Those are out-\(\Phi_i\) lined below.

The complex amplitude of the heat loss of an \(i\) indexed room to all the neighbouring rooms can be described as:

\[
\Phi_i = - \sum \tilde{L}_{i,j} \cdot \hat{\Theta}_j
\]

(1)

The exterior is involved into this model in a kind of “external rooms”. Thus, there is a linear relation between the complex amplitude of the heat loss \(\Phi_i\) of the room \(i\) and the complex amplitudes of the transient behaviour of the air temperatures \(\hat{\Theta}_j\) of all the rooms adjacent to the construction. The complex harmonic thermal conductances \(\tilde{L}_{i,j}\) act as proportionality factors. In equation (1), it has to be summated over all the rooms, including the room \(i\) itself. This approach (1) is part of the international standard EN ISO 13786 [6] for the special case of constructional components that are adjacent to two rooms only (“interior”
and “exterior”). It is the basis for the definition of the effective heat storage capacity\(^1\). The steady-state, i.e. time-independent case relation (1) can be described as [2]:

\[ \Phi_i = -\sum_j L_{i,j} \cdot \Phi_j \]  

(2)

\(\Phi_i\) and \(\Phi_j\) are the zeroth harmonic of their respective Fourier-Series and are real numbers (the mean value of the heat loss and the mean value of the air temperature profiles). In the steady-state case, the thermal conductances \(L_{i,j}\) are real numbers as well.

Due to energy conservation, the steady-state case (only) is:

\[ \sum_j L_{i,j} = 0 \]  

(3)

By using relation (3), equation (2) can be altered to:

\[ \Phi_i = \sum_{j \neq i} L_{i,j} \cdot (\Phi_i - \Phi_j) \]  

(4)

This basic relation of the theory of thermal conductances is part of the standard EN ISO 10211 [7]. According to [1], the calculation of temperature distribution inside a construction results in a linear relation as well. The complex amplitude \(\hat{\Theta}\) of the temperature at any point of a construction can be determined by using:

\[ \hat{\Theta}(x, y, z) = \sum_j \hat{g}_j (x, y, z) \cdot \hat{\Theta}_j \]  

(5)

Thus, \(\hat{\Theta}(x, y, z)\) is linearly dependent on the complex amplitudes of the air temperature profile \(\hat{\Theta}_j\) of each of the rooms adjacent to the constructional component. The complex dimensionless harmonic temperature-weighting-factors \(\hat{g}_j (x, y, z)\) act as proportionality factors. Again, it has to be summated over all the rooms adjacent to the construction using equation (5). For the zeroth harmonic, that is the steady-state case, equation (5) keeps its structure:

\[ \Theta(x, y, z) = \sum_j g_j (x, y, z) \cdot \Theta_j \]  

(6)

The temperature-weighting-factors \(g_j\) are real numbers in case of the zeroth harmonic. \(\Theta_j\) are the mean values of the temperature profiles of each of the rooms adjacent to the construction. \(\Theta\) is the temperature on the position marked by \((x, y, z)\).

Like the thermal conductances and the temperature-weighting-factors, the harmonic thermal conductances and the harmonic temperature-weighting-factors are independent of the air temperatures as well. That is why they are ideally suited for acting as constructional indicators. They can be calculated by suitable thermal bridge analysis software without the need of defining borderline conditions. According to EN ISO 10211 [7], the steady-state conductances and the temperature-weighting-factors state the result of a thermal bridge calculation as significant indicators.

At given conductances and temperature-weighting-factors, the thermal conduction and heat storage processes can be described three-dimensionally for variously shaped and

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\(^1\) Despite several requests to correct, there is still a sign-error in equation (4) of EN ISO 13786.
arbitrarily composed constructions – by using the basic relations of (1), (4), (5) and (6). There is no further limitation regarding the amount of interior and exterior rooms adjacent to a construction. The construction may be a section of the building envelope or the entire building envelope; even the totality of all the components of a building can be considered to be a construction.

The following procedure is required for the calculation of the temperature profile and/or the heat flows. In the first step, the mean values and complex amplitudes of given temperature profiles and/or a given heat flow profile are calculated using Fourier-Analysis. The amount of harmonics depends on the question to be answered and the accuracy requirements of the result. In the second step, the complex amplitudes of the variable to be found are calculated for each harmonic by using the relations of (1) and/or (5). The likewise required mean values can be easily calculated by equations (4) and (6). Obviously, the thermal conductances and the temperature-weighting-factors have to be determined beforehand using a periodic thermal bridge calculation. The final Fourier-Synthesis leads to a parameter varying in time.

For instance, the temperature profile of any point of a constructional component can be calculated in this way. Thus, the temperature field of a three-dimensional constructional component can be calculated, including its changes in time. Likewise the time-varying heat losses of each room adjacent to a constructional component can be determined.

3. The visualization of the calculation results

Three-dimensional, periodic calculation on the basis of the conductance theory is already implemented in the thermal bridge program AnTherm [3]. While implementing the calculation module, the visualization of the calculation results turned out to be a special challenge. Since the calculation result is dependent on four dimensions which are three spatial coordinates plus time, it cannot be visualized right away. However, there are various possibilities of showing the calculation results by using appropriate visualizations in a smaller dimension. The applicable choice of visualization highly depends on the question to be answered.

Even in a steady-state case, it is difficult to visualize the calculated temperature distribution in three-dimensional models clearly. To avoid this problem, it is possible to reduce the dimension by laying flat sections through the construction. On these sections, the calculated temperature distribution can be shown by using a false-colour image or/and isotherms. Moving the section plane along an axis gives the impression of temperature distribution inside a constructional component.

By calculating the temperature distribution on a specific section plane for a certain point of time, it is now possible to show it as a slide show. This type of visualization may give the user an impression of the effects of heat conduction and heat storage inside a component; however, it is very unlikely that this will lead to usable quantitative conclusions.

In order to obtain more usable information, it is necessary to reduce the visualization to two dimensions. Through this kind of visualization, most questions will be answered in the best manner. An example may be heat loss varying in time from one room to another or to the outside. In this case, the result of a three-dimensional, transient simulation can be easily presented by means of diagrams.
Regarding the temperature distribution, it is often more useful to focus on specific points than try to visualize it in a more complicated way. Thus, according to EN ISO 10211 [7], it is reasonable to check the point of minimal surface temperature in each room. However, a search for this point, regarding the transient case, is more complicated. Beforehand, minimum temperature profiles of each point have to be determined and compared.

The totality of points of equal temperature is another very important task in practice. One possibility to visualize the result of this question is to draw isotherms on cutting planes which run through the model at a certain time. Sometimes it is more useful to visualize an isosurface in a three-dimensional model at a specific time.

An example of this matter would be the determination of the frost line in the ground close to buildings in order to be able to protect a frost-free foundation.

4. An example of application

While assessing the thermal performance of a building envelope, a description of heat losses of earth-coupled components takes a special position for two reasons:

1. A one-dimensional approach, that is often used for air-touched components to calculate partial conductances by multiplying heat-transfer coefficients and the respective component surface areas, would not make any sense. The two-dimensional calculation of heat loss does not bring any useful results of practical relevance because the influence of the corners cannot be ignored [8].

2. In connection with heat loss through earth-coupled components, wide areas of the ground surrounding a building are affected by heat flow.

Most steady-state or quasi-steady-state calculations neglect the effects of heat storage. This leads to major misjudgments because of the strong influence of heat storage [9]. Therefore, the thermal behaviour of earth-coupled components has to be calculated in a transient way. In most cases, the thermal behaviour of a building over the long time average is of interest. This means that the periodic steady-state calculation approach on the basis of the annual period is perfectly suited for describing the thermal behaviour of earth-coupled components. For this reason, the foundation slab of a detached house in the passive standard will serve as an example of three-dimensional periodic calculation in the following section.

A square cutout of the foundation slab, including one metre of the rising external wall, falls under consideration. Figure 1 shows the structure and the three-dimensional calculation model. Besides the construction of interest, according to EN ISO 10211, large areas of the surrounding soil are included in the model.

As a result of three-dimensional periodic calculation, AnTherm [3] provides an immediate thermal conductance between the inside and the outside as well as the matrices of the harmonic thermal conductances for the six considered harmonics. The heat loss through the section in the course of the year, as shown in Figure 1, is calculated by the three-dimensional, periodic based thermal building simulation program Thesim [10] using equations (1) and (4). Assuming the constant inside air temperature of 20°C and a long-time, smoothed annual profile of the outside temperature for the meteorological station Hohe Warte in Vienna results in the annual course of heat loss output as shown in Fig. 2.
In addition to the correct evaluation of heat loss through earth-coupled components, the frost line in the ground plays an important role. The calculation of heat loss, based on long-term mean values, cannot be used to ensure a frost-free foundation. To obtain realistic results, it is necessary to apply extreme winter conditions. The presented case is based on the year with the most extreme winter outdoor temperatures out of a 50-year series (1960-2009, Vienna, Hohe Warte). 1966 – the only year with monthly mean values below the freezing point for December, January and February – was chosen.

The following figure shows the calculated temperature field on the section plane running through the exterior wall and the foundation slab for February 8th, 8:00. The figure on the right side of Fig. 3 provides more information about the frost-resistance of the structure located in Vienna.

**Fig. 1. Construction and calculation model**

Rys. 1. Szkic konstrukcji i model obliczeniowy

**Earth-Coupled System [Slab]**

**Fig. 2. Annual course of heat loss [10]**

Rys. 2. Roczny przebieg strat energii cieplnej [10]
It turns out that a strip foundation remains frost-free in the area of the selected section even in extreme winter frost. The question whether the foundation remains frost-free in the area of the edge of a building has not been answered yet. This is revealed by the visualization of the isosurface for 0°C in the following figure.

The figure above shows that the foundation keeps frost-free for the Viennese site and locations with similar exterior climatic conditions. While interpreting this result, it has to be considered that AnTherm [3] uses non-time-varying material parameters. The effects of moisture transport and ground freezing are neglected. Analyzing the influence of these effects on the temperature field [11] in detail shows that the consideration leads to slightly elevated temperatures. Thus, the temperature fields calculated by AnTherm [3] are on the safe side.

Fig. 3. Isotherm visualization of temperature distribution on February 8th, 8:00, section plane running through external wall and foundation slab. Right - zero-grade isotherm

Rys. 3. Wizualizacja rozkładu temperatury (izotermy) w dniu 8 lutego, 8:00, przekrój poprzez ścianę zewnętrzną i płytę fundamentu. Z prawej - izoterma dla temperatury 0°C

Fig. 4. Isosurface for 0 °C, February 8th, 8:00

Rys. 4. Izopowierzchnia dla temperatury 0°C w dniu 8 lutego, 8:00
References