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DETERMINATION OF VERTICAL AND HORIZONTAL SOIL DISPLACEMENTS IN AUTOMATED MEASURING SYSTEMS ON THE BASIS OF ANGULAR MEASUREMENTS

WYZNACZANIE PIONOWYCH I POZIOMYCH PRZEMIESZCZEŃ GRUNTU W AUTOMATYCZNYCH SYSTEMACH POMIAROWYCH NA PODSTAWIE POMIARÓW KĄTOWYCH

A b s t r a c t

The increasing interest in monitoring systems for soil displacements, prompted the authors to search for calculation methods which would allow the construction of monitoring devices without the need to place sensors in inclinometric tubes. The application of the spline interpolation method and the local approximation method by means of weighted moving squares allowed for the creation of curves which describe the soil deformation with the required accuracy. The basic equations of this calculation method and numerical examples are presented in the paper.

Keywords: *soil deformation measurements*, *inclinometers*, *spline interpolation*

S t r e s z c z e n i e

Wzrastające zainteresowanie systemami monitorowania przemieszczeń gruntów skłoniło autorów do poszukiwania metod obliczeniowych, z wykorzystaniem których możliwe byłoby budowanie urządzeń pomiarowych bez konieczności zabudowy czujników w rurach inklinometrycznych. Zastosowanie metody interpolacji składanej oraz metody lokalnej aproksymacji za pomocą techniki ważonych ruchomych kwadratów umożliwiło zbudowanie krzywych opisujących deformację gruntu z założoną dokładnością. W artykule przedstawiono podstawowe równania metody obliczeniowej oraz przykłady numeryczne.

Słowa kluczowe: *pomiary deformacji gruntu*, *inklinometry*, *interpolacja sklejana*

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1. Introduction – contemporary design tools

Currently, linear objects are very often located on soils of a high compressibility since terrains of more favourable geological structure are the most often taken by building and other locations are not possible. This causes the necessity to apply various geotechnical methods leading to the strengthening of the subsoil. Often, the economically favourable solution is making an overloaded embankment. Obtaining the soil improvement effect is in this case related to the consolidation of the soil. The consolidation effect is based on the reduction of the soil volume as a result of water outflow and decreasing distances between soil particles. The duration time of the consolidation process depends, first and foremost, on the infiltration coefficient c_v and the load value resulting from the built overload embankment. It has been observed in accurate investigations [1] that increased stress in the soil causes a certain reduction of the consolidation coefficient. This effect, as well as the natural variability of soils, is the reason that the theoretically determined settlement of the improved base and its minimal time can differ from the real situation. Therefore, during the consolidating improvement process performing its constant control is recommended. Apart from the measurements of water pressure changes, the basic parameters describing the effect of soil improvement are vertical and horizontal displacements. Measurements of vertical displacements allow for tracing the consolidation process over time, while horizontal displacement measurements enable controlling the effect of soil driven out from strengthened spaces. More and more often, the automatic monitoring systems which allow for observation of soil displacement both at the stage of object building and during operation, are used for these measurements.

2. Measurement methods of vertical and horizontal soil displacements

The measurement of vertical soil displacements is usually perceivable as measurements of its surface deformation. Broadly understood geodetic surveying methods from the classic levelling, via satellite techniques GPS [2], up to laser scanners, are here applied. However, with regard to the consolidation process, it is essential to observe the soil response to the overload applied in the space where displacements occur, meaning within the improved soil.

The basic way of measuring vertical soil displacements is through the use of plate benchmarks [3]. This measurement allows for determining the soil displacements in the measuring points and at time intervals at which observations are made. More and more often, hydraulic prophilometers, manual or automatic, are used for measuring the vertical soil displacements. Hydraulic prophilometers are devices in which the sensor measures the hydrostatic liquid pressure change caused by changes of the measured value in relation to the assumed reference point. Vertical soil displacements in the selected places can also be measured by means of indenting extensometers or individual points of hydraulic measurement.

The best known and the most commonly applied method is taking inclinometric measurements [4]. These measurements allow for determining vertical as well as horizontal displacements. This method is used both for manual measurements and for building automated measuring systems tracing the consolidation process.

3. Inclinometric measurements

Inclinometric measurements are based on determining shape changes of the inclinometric tube placed in the soil due to the ground deformation. The measurement is done by means of the probe called the inclinometer. The measurement can be done manually, by an individual probe shifted along the inclinometric tube or automatically by stable inclinometers (a few or several dozen inclinometers connected together). Angular positions *n* of measured segments (probes) of a known length *L* are determined (Fig. 1). These items of information are then recalculated into vertical or horizontal displacements, depending on the kind of task. The input data are as follows:

– total length, at which the measurement is done, L_t,
– number of measuring sensors (segments, probes, protractors) *n*, connected by articulated joints of length *L*: ($L = L_t / n$),
– angles of inclination of individual probes α_i, $i = 1,\ldots, n$, (acc. to Fig. 1), in their middle points,
– reference height *d* (height of the first point of the segment).

Measuring sensors determine points of the broken line situated on the original curve. Coordinates can be calculated from the equation:

$$\begin{cases} x_i = L\cos(\alpha_1) + L\cos(\alpha_2) + \ldots + L\cos(\alpha_{i-1}) = L\sum_{j=1}^{i-1}\cos(\alpha_j) \\ d_i = d + L\sin(\alpha_1) + L\sin(\alpha_2) + \ldots + L\sin(\alpha_{i-1}) = d + L\sum_{j=1}^{i-1}\sin(\alpha_j) \end{cases}, \quad i = 1,\ldots,N \tag{1}$$

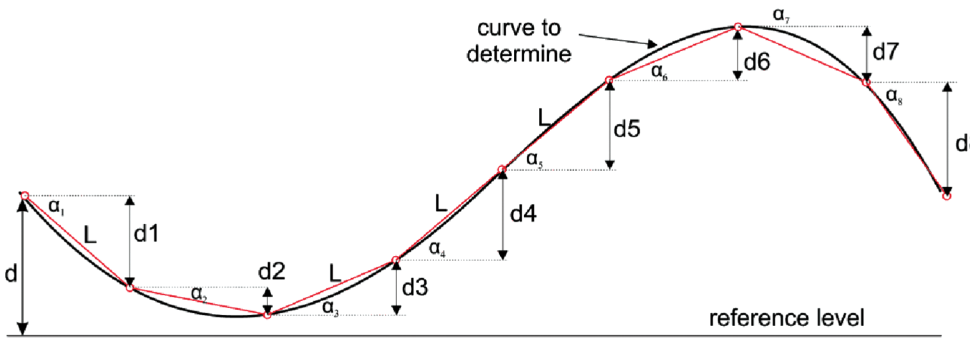

Fig. 1. Principle of inclinometric measurements

This solution (later referred to as the ‘simplified solution’) allows for obtaining the displacement in an arbitrary point of the segment by connecting points of coordinates (1) with straight segments (broken line). Then, the vertical soil displacement function can be written in one equation, joining with each other straight lines in individual segments:

$$d(x) = a_1 x + a_2 + \sum_{i=2}^{N-1} b_i \left(x - x_i\right)_+ , \quad \left(x - x_i\right)_+ = \begin{cases} x - x_i, & \text{for } x > x_i \\ 0, & \text{for } x \le x_i \end{cases} \tag{2}$$

where:

$$a_1 = \frac{d_2 - d_1}{x_2 - x_1}, \quad a_2 = d_1 - a_1 x_1 \tag{3}$$

and:

$$b_j = \frac{d_{j+1} - \left(a_1 x_{j+1} + a_2\right) - \sum_{i=2}^{j-1} b_i \left(x_{j+1} - x_i\right)}{x_{j+1} - x_j}, \quad j = 2,3,\ldots,N-1 \tag{4}$$

Points (1) are lying on the original curve, with the accuracy determined by the protractor measuring precision (measuring range). In spite of the simplicity of such an approach, the solution does not correspond with real soil deformation. It can be expected that the largest errors will occur in the middle of the measuring segments.

4. Inclinometric measurements in automated monitoring systems

Inclinometric measurements can by easy read autamatically. Sets of stable inclinometers are placed in previously prepared vertical measuring openings or in horizontal tubes. After connecting them to the device reading the data, the automatic monitoring system is ready – this allows for a very fast measurement of the soil displacement profile. The read-out of all sensors from the given measuring cross-section requires only a few seconds. This provides the possibility of tracing, for example, the consolidation process and the soil displacement from under the over-loaded embankment during building works and adjusting their successive stages to the progressing consolidation process. Automatic systems can be equipped with additional sensors, e.g. of the water pore pressure measurements.

High equipment costs constitute a significant limitation in the inclinometric measurements automation. The measuring probe length is usually 1 meter. This means that, for the profile 30 meters long, the measuring chain should consist of 30 sensors. Additionally, for communicating with sensors, a recording device of 30 channels is needed. One of the ways of decreasing the costs of the monitoring system can be the elongation of measuring probes. However, this operation has physical limitations relating to the diameter of standard inclinometric tubes and to the bending stiffness of the measuring probes.

To avoid these limitations, it is possible to place special probes directly into the soil. However, in this case, it is necessary to take into account length changes of the measuring set due to soil deformation. When inclinometric tubes are applied, the above problem does not exist since the measuring chain retains its length displacing itself inside the tube. Length changes of the measuring chain do not allow the application of simple equations for the measured results analysis.

Thus, there is a need for development of the measuring method which would allow the above mentioned limitations to be overcome. Information on an angular position of each measuring segment must be recalculated into vertical soil displacements in a way which would enable making the automatic measuring system without the need for protecting tubes.

5. Complex calculation methods of soil displacement and assessment of their accuracy

The problem of identifying vertical soil displacement on the basis of data concerning angular positions of measuring segments requires solving the mathematical approximation task [6], [7], i.e. in general cases, fitting the curve to the measured data.

Defined in such a way, the complex solution not only better describes the real soil movements after deformation, but can also serve as the accuracy indicator of the obtained previously simplified solution (1). This solution will be determined in the form of a curve of the proper order (e.g. multinomial), Fig. 2, which will be controlled by fitting to the measuring points of coordinates:

$$\begin{cases} x_i = \dfrac{L}{n}(i-1) \\ d_i = L\mathrm{tg}(\alpha_1) + L\mathrm{tg}(\alpha_2) + \ldots + L\mathrm{tg}(\alpha_{i-1}) = L\sum_{j=1}^{i-1}\mathrm{tg}(\alpha_j) \end{cases}, \quad i = 1,\ldots,N \tag{5}$$

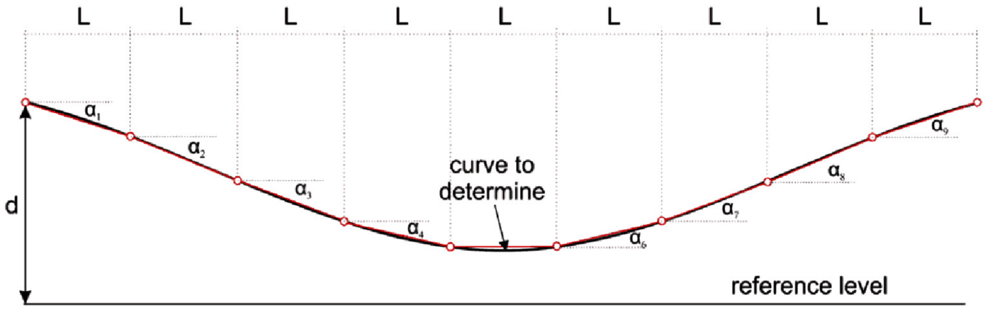

Fig. 2. Principle of determining the subsidence curve by means of the complex solution

The simplest approximation, in the result of which the curve will be obtained, is the interpolation [5], [7]. The interpolation simplicity relies on the fact that the curve passes through all given points and that this is the only condition which has to meet. Out of various interpolation techniques, the most often the so-called Lagrange's interpolation formula is applied:

$$d(x) = \sum_{i=1}^{N} d_i L_i(x), \quad L_i(x) = \prod_{\substack{j=1 \\ j \neq i}}^{N} \frac{x - x_j}{x_i - x_j} \tag{6}$$

However, this formula has a significant limitation. In a similar fashion, as in each multinomial interpolation, at a large N, the solution in between points can become unstable, especially at the segment ends. This phenomenon is called Runge's effect and most often occurs in cases when, for large numbers of initial data, the original function (given either in a continuous or discrete way) indicates large differences in values (large gradients). Therefore, in practice, the applicability range of equation (6) is also limited.

Another interpolation technique applied here is the so-called spline interpolation [7], in which curves of low orders (most often 2 or 3) are applied on individual segments which are then splined together in such a manner as to retain the continuity of the function and its successive derivatives (determined by the curve order). Its certain variant is the broken line (2), for which only the function continuity is ensured. In the case of the spline square interpolation, the equation for the displacement can be written as:

$$d(x) = a_1 x^2 + a_2 x + a_3 + \sum_{i=2}^{N-1} b_i \left(x - x_i\right)_+^2, \quad \left(x - x_i\right)_+^2 = \begin{cases} \left(x - x_i\right)^2, & \text{for } x > x_i \\ 0, & \text{for } x \le x_i \end{cases} \tag{7}$$

where coefficients a_1, a_2, a_3 are the solution to the set of equations:

$$\begin{cases} d(x_1) = d_1 \\ d(x_2) = d_2 \\ d'(x_1) = \operatorname{tg}\left(\alpha_1\right) \end{cases} \Rightarrow \begin{cases} a_1 x_1^2 + a_2 x_1 + a_3 = d_1 \\ a_1 x_2^2 + a_2 x_2 + a_3 = d_2 \\ 2a_1 x_1 + a_2 = \operatorname{tg}\left(\alpha_1\right) \end{cases} \Rightarrow \begin{cases} a_1 = \ldots \\ a_2 = \ldots \\ a_3 = \ldots \end{cases} \tag{8}$$

And coefficients b_i, $i = 2, 3, \ldots, N-1$ are resulting from the following recurrence formula:

$$b_j = \frac{d_{j+1} - \left(a_1 x_{j+1}^2 + a_2 x_{j+1} + a_3\right) - \sum_{i=2}^{j-1} b_i \left(x_{j+1} - x_i\right)^2}{\left(x_{j+1} - x_j\right)^2}, \quad j = 2, 3, \ldots, N-1 \tag{9}$$

The application range of equation (7) is wider than of equation (6). Solutions by means of spline functions are worth being applied when the measurement data are numerous ($N >> 6$), and the soil deformation is relatively mild (modelling of depressions, hillsides).

The last method discussed here is the interpolation method, this is a local approximation method by means of the weighted moving squares technique [6]. In a similar fashion as in the case of spline curves, local multinomial curves of low orders are used, however, they are ascribed to points not to segments, and their passing through the given points is controlled by weighted function. Its singularity in the given points assures interpolation properties of the final curve in spite of coefficients selected by approximation (minimisation of errors understood as the weighted sum of squares of deviation values on the curve and the given value).

The local curve (multinomial of a low order) is built on a group of a few points from the nearest vicinity of the point for which the approximation value is needed. A group of such points is called a star. The number of star points (m) depends on the task dimensionality and on the curve order. For the parabola ($p = 2$) at least 3 points should be taken, but no more than 6–10 (for more points, the curve loses the local character). In order to determine the displacement of the arbitrary point x, the following matrix values should be calculated:

$$\underset{[m\times s]}{\boldsymbol{P}(x)} = \begin{bmatrix} 1 & h_1 & \frac{1}{2}h_1^2 \\ 1 & h_2 & \frac{1}{2}h_2^2 \\ \dots & \dots & \dots \\ 1 & h_m & \frac{1}{2}h_m^2 \end{bmatrix}, \quad \underset{[m\times m]}{\boldsymbol{W}(x)} = \begin{bmatrix} \omega_1(x) & 0 & \dots & 0 \\ 0 & \omega_1(x) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \omega_m(x) \end{bmatrix} \tag{10}$$

$$\underset{[s\times m]}{\boldsymbol{M}(x)} = \left(\boldsymbol{P}(x)\boldsymbol{W}(x)\boldsymbol{P}(x)\right)^{-1}\boldsymbol{P}(x)\boldsymbol{W}(x)$$

where s – number of approximation parameters ($s = p + 1$), $h = x_i - x$ and:

$$\omega_i(x) = \frac{1}{|h|^{2(p+1)} + \varepsilon} \tag{11}$$

ε is a very small number, corresponding to the calculating accuracy (e.g. $\varepsilon = 10^{-16}$ for the real type of dual precision). Then, the displacement is expressed by the equation:

$$d(x) = \sum_{i=1}^{m} M_{1,i}(x)d_i \tag{12}$$

This method can be successfully applied in cases where there are a lot of measuring results and the soil deformation is highly variable (e.g. in the case of a fault or hole). A certain difficulty in its application can constitute the fact that the analytical form of the curve is unknown – its calculation by a point after point according to equations (10)–(12) is possible.

Each of the complex equations (6), (7) and (12) can be used for the assessment of the simplified solution quality (2). A difference between the simplified solution and one of the complex solutions should indicate the true error of the simplified solution, which for real measurements, will not be known. Whereas the error of complex solutions can be assessed in a few ways. Analogous to the assessment of the simplified solution error, curves of higher orders than in the case of (7) and (12) can be built. It is also possible to retain the same order (second), but instead to build interpolations on a larger number of data. In this case, angular measurements can be supplemented by additional virtual data calculated on the basis of the complex solution.

6. Numerical examples

With the aim of testing the measuring data, at the length L_t = 30 m, were simulated by means of curves not being multinomial (of ordinates from the range ± 0.5 m). Calculated angles were additionally disturbed within limits of the measuring accuracy of protractors (± 0.1% of the measuring range (–10° 10°) i.e. ± 36”). The data prepared in such a way were used in controlling calculations.

Three commonly occurring types of soil deformations, differing in their forms and in number of extreme points, were subjected to analysis. Results of calculation for each type of deformation are shown in the figure (Figs. 3–5). The calculations were performed for the configuration of n = 10 or 30 inclinometers (of the same length).

The upper diagram (Figs. 3–5) contains:

– accurate solution (on which angles were found) – dashed blue line,
– simplified solution (2) – dotted pink line,
– one selected complex solution (typical for the given deformation form) – solid red line.

The lower diagram (Figs. 3–5) contains solution errors and their assessments:

– accurate error of the simplified solution (difference between the simplified solution and the accurate mathematical solution) – dotted pink line,
– estimated error of the simplified solution – dotted green line,
– accurate error of the complex solution – dashed blue line,
– estimated error of the complex solution – solid red line.

Maximum displacement values of the module and errors are given in diagram headings. The first case constitutes a large, basin type soil subsidence, which may be modelled by equation (6) or (7). Equation (7) and a configuration of 10 inclinometers of a length L = 3 m were used in calculations. The results are presented in Fig. 3.

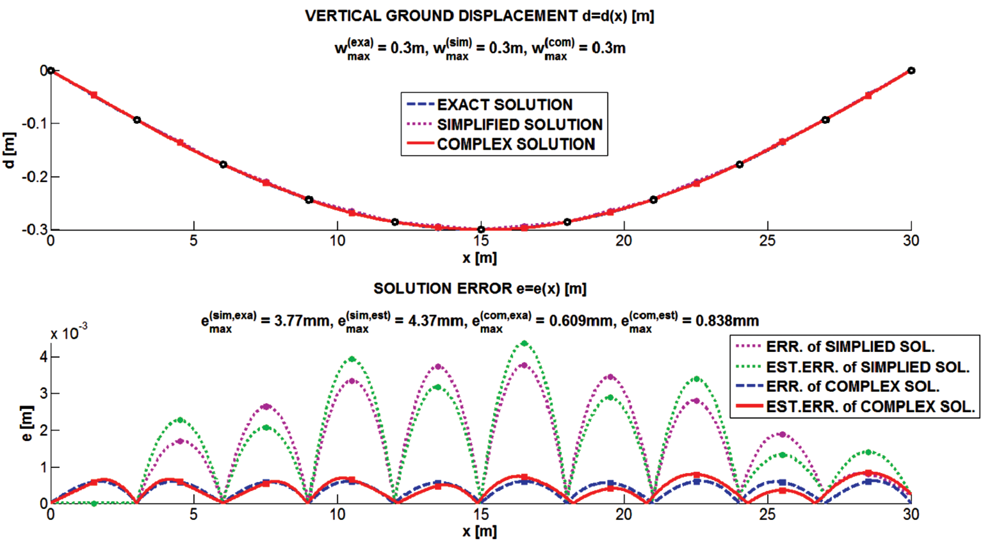

Fig. 3. Calculation results for the first form of the soil deformation (configuration of 10 inclinometers)

The error of the complex solution is one order smaller, and the solution successfully estimates the simplified solution error.

The second case constitutes the slope and its model will be the spline square interpolation, calculated according (7). The results, also for n = 10, are presented in Fig. 4.

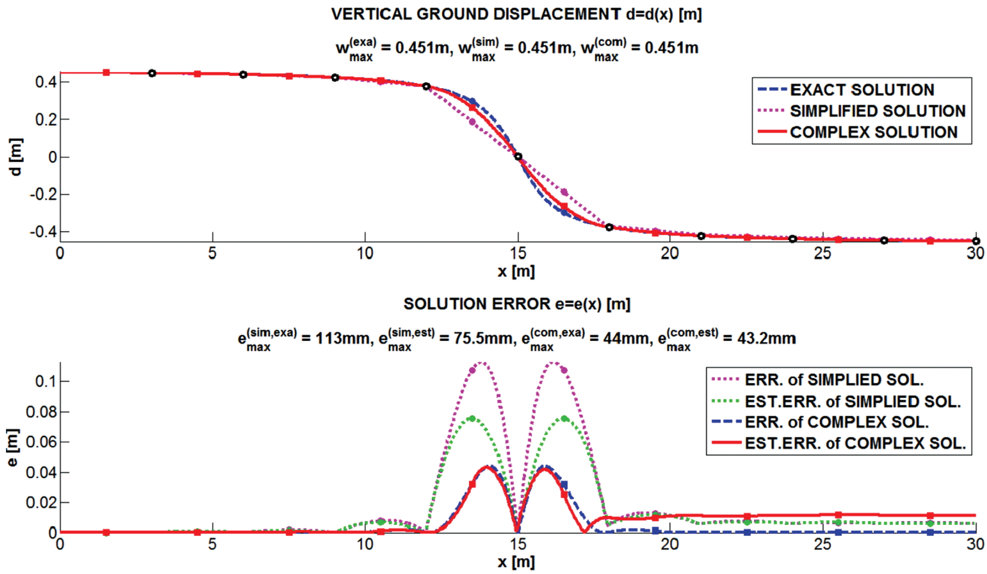

Fig. 4. Calculation results for the second form of soil deformation (configuration of 10 inclinometers)

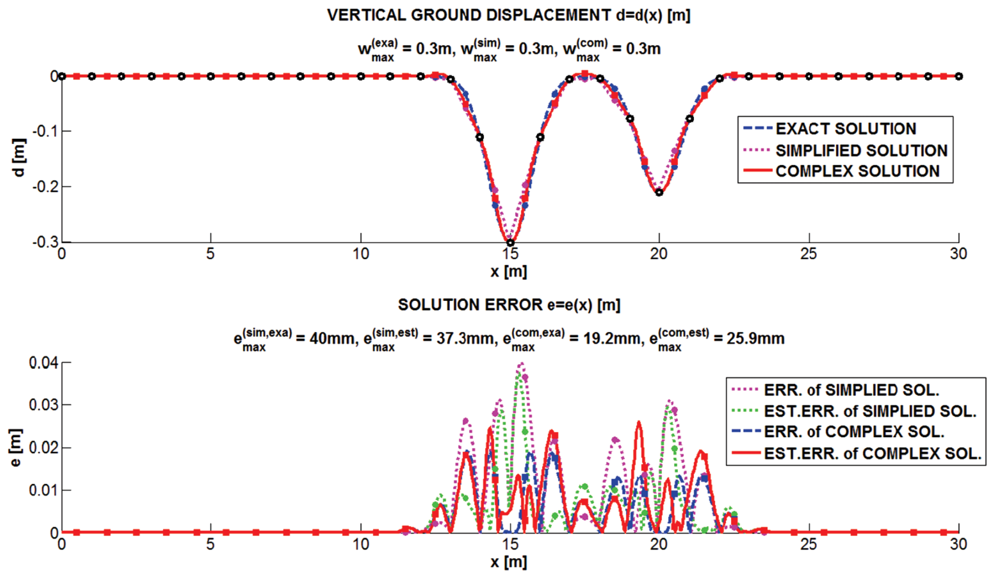

Fig. 5. Calculation results for the third form of soil deformation (configuration of 30 inclinometers)

The last case (3rd) constitutes two crater type depressions of different depths. The configuration of $n = 30$ inclinometers with a length $L = 1$ m each, and the local weighted approximation (10)–(12) were used in calculations. The results are presented in Fig. 5.

The estimated errors can be used not only in statements of how accurate was the solution, but also, for example, the optimal planning of numbers and placements of measuring sensors. For the second calculation example there is a need for an increased number of sensors in the middle zone and of decreased outside it. One of the plans for the discussed system is checking its usefulness for the dynamic measurements of bridges.

7. Conclusions

The presented method of analysis of data obtained from the measurement chains built from inclinometric rods allows:

- improvement of the result accuracy in cases of applying the classic inclinometric measurements,
- performance of measurements of soils displacements by means of measuring chains, without protecting tubes.

Renouncement of using inclinometric tubes, the method allows the building of automated systems of significantly long measuring elements, and to achieve sensible costs of building the measuring system.

Two methods of fitting curves to measured data were presented and discussed here. The commonly applied simplified technique is based on the assumption of there being straight segments between the measuring sensors, constituting the straight broken line (first order spline function). Though it interpolates data points, its accuracy is usually low, with the highest errors located between the sensors. The second, more complex approach, is based on mathematical approximation methods, like Lagrange interpolation, spline functions of the higher orders (2, 3), and the most general method, local approximation generated by moving the least squares. The numerical tests performed clearly showed that these methods reproduce the actual solution (soil displacement) in a much more accurate manner, especially in cases where solution gradients are high (ground collapse or hillsides). Moreover, these solutions may be applied as the error estimators for the simplified solution, if necessary.

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