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THE RANDOM VARIABILITY ANALYSIS OF THE MECHANICAL PROPERTIES OF THE SELECTED ALUMINUM ALLOYS

ANALIZA ZMIENNOŚCI LOSOWEJ CECH MECHANICZNYCH WYBRANYCH STOPÓW ALUMINIUM

A b s t r a c t

This paper presents the results of the experimental research on mechanical properties of selected aluminum alloys and also statistical methods for the analysis of the random variability. Results of laboratory tests describe the local changes of the strength characteristics of Re, Rm and Young's modulus E.

Keywords: *aluminum, testing, homogeneity, variability, analysis,*

S t r e s z c z e n i e

W artykule zaprezentowano wyniki własnych badań cech mechanicznych wybranych stopów aluminium oraz metody analizy statystycznej ich zmienności losowej. Otrzymane rezultaty badań opisują przebieg zmian lokalnych cech wytrzymałościowych R_e, R_m i E wzdłuż osi badanych prętów.

Słowa kluczowe*: aluminium, badania, jednorodność, zmienność*

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1. Introduction

The main goal of the study is the analysis of variability and identification structure for the sparklines of the mechanical properties of selected aluminum alloys. The mechanical properties of aluminum alloys depend on the content of alloying elements, as well as on the type of thermal or mechanical treatment. Effect of tempering the strength of the alloy is a specific feature of the aluminum alloy, which can significantly affect the uniformity of strength properties along the axis of the bar.

Studies of variability of the mechanical properties along the axis of bars have been carried out only on specimens of steel. Difference in the cooling rate of steel rods stored in circles, according to a study [1] and also [2] resulted in differentiation of the rods strength through the length, as demonstrated by the occurrence of a trend. Metallurgical products used in the steel or aluminium structures have lengths of a few to several meters, the assessment of the variability of local strength properties of steel such St_3S was carried out in [3]. Obtained series of values f_y and f_u – *c*onsidered as realizations of a random function, were characterized by random noise. Based on knowledge of production technology aluminum alloys, cannot be excluded that it is non-stationary process. Random variability of material properties affects the variability of the whole structure, and therefore, the statistical distribution, found in the researches of the material, has a significant impact on the ability of the structure to carry loads.

2. The issue of stochastic homogeneity of the material

The concept of stochastic homogeneity is introduced when all the elements of the set (aluminium product) correspond to random variables or random functions $Y\,(f_y, f_u, E)$ with the same distribution. However a diversity distribution of random variables Y and non-stationarity random function Y is stochastic heterogeneity. The stochastic process $F(t)$ is called a stationary process if its properties do not change when moving the timeline, the mean and variance is constant and the correlation between the cross-sections of the process $F(t)$ and $F(t + \Delta t)$ depend only on the distance Δt, and not t. With regard to the local characteristics of strength over the length of the rod, the time t is replaced by a measure of length. The non-stationarity can provide signals such as deterministic component of an unknown course and difficult to determine the analytical, the presence of a trend – deterministic component linearly variable along the length of the rod, or a harmonic of the period.

To test the strength characteristics of local changes in the length of the rod, carried out the static tensile test, the results are attributed to the longitudinal axis of the rod points. The local yield strength R_e, identified with the result of the tensile test section of the rod of length L_0, is determined for the area Ω (metallographic grain size) on the order of a few centimeters, in the central plane of the bar [3]. After bonding to yield a succession of n sections of the rod are obtained argument continuous random function $R(x)$ for $x \in \langle 0,L \rangle$. $R_j(x)$ as a single realization, characterized by a hypothetical general population associated with a single rod, coming from one production cycle [2, 3].

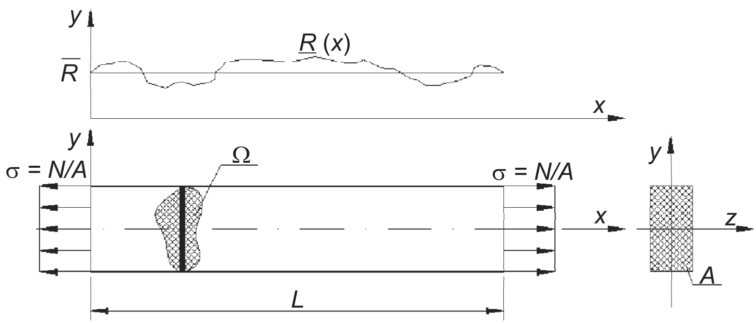

Fig. 1. The concept continual model of bar tensile strength (source: [3] fig. 4.16)

Verification of uniform sets of results is based on statistical tests. The study of statistical features is performed in two stages: putting statistical hypothesis for a single implementation and its verification by an independent statistical material.

3. Research of variation of mechanical properties of aluminum alloys

Experimental research on random variation of mechanical properties of aluminum bars included two parts of aluminium alloys in series 6xxx (durability class B) and 5xxx (durability class A) in a single product.

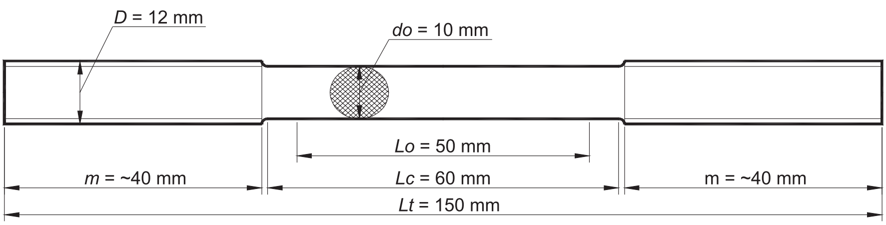

Fig. 2. The proportional round specimen

The figure Fig.1 shows specimen, which was adopted to tensile test. The specimens were cut from round bars with a diameter D = 12mm aluminum alloy AW-6060 T6 (3 bars in length 6m, signed as A, B and D) and AW-5754 H14 (4 bars in length 3m, signed as E,F,G,H).

3.1. The static tensile test

Mechanical properties of aluminium alloys were performed by static tensile test using electro-mechanical testing machine Zwick, equipped with extensometer, with a measuring range 2,4 kN to 1200 kN for Class 1 (see Fig. 3), using computer-aided measuring system.

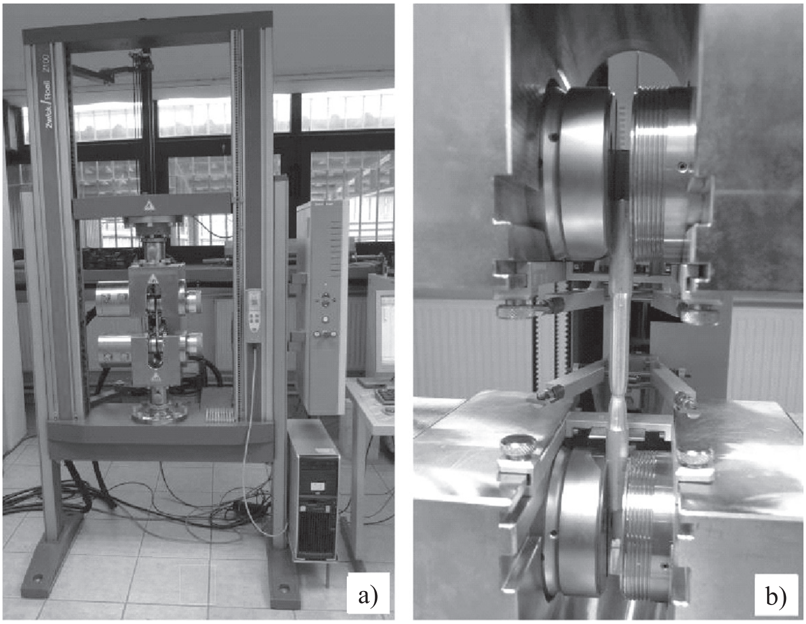

Fig. 3. The static tensile test: a) the testing machine Zwick Roell Z100, b) proper placement of the specimen in the grips (source: [11])

The study was conducted at ambient temperature and relative elongation rate of permanent deformation of specimens and the measuring range of the testing machine determined according to the standard EN 10002–1:2002 [4].

3.2.Test results

As a result of the static tensile test results were obtained for the following sequences of specimens: diameter measurements, yield strength, ultimate strength and Young's modulus. The diameters of the specimens were measured in two perpendicular directions. The diameters (Table 1) used for the analysis was the average of the two measurements in three sections (in two perpendicular directions) on the base length of each sample.

Table 1

Summary of the results diameter measurements for bars in alloy EN-AW 6060

Bar	n	Mean $\bar{x}$	Variance $s2$	Standard deviation s	Coeff. of variation v [%]
Diameter D [mm]					
A	37	10,015	0,013	0,011	0,11
B	38	10,015	0,020	0,014	0,14
D	36	10,013	0,010	0,010	0,10

Diameter measurements were performed to assess the uniformity of the control bars batch by analysis of variance. Analysis of results of measurements of diameters showed a slight variation, therefore, further analysis does not take into account the effect of the diameter of the specimen on the results of static tensile test.

The graphs (Fig. 4) shows the realizations of yield strength $f_{0,2}$ for 38 specimens cut from the bar A, 38 from bar B and 36 from the bar D and (Fig. 5) 19 specimens cut from the bar E, 17 from the bar F,18 from the bar G and 19 from bar H. Red circles denote values considered as outliers in carrying out the appropriate statistical tests.

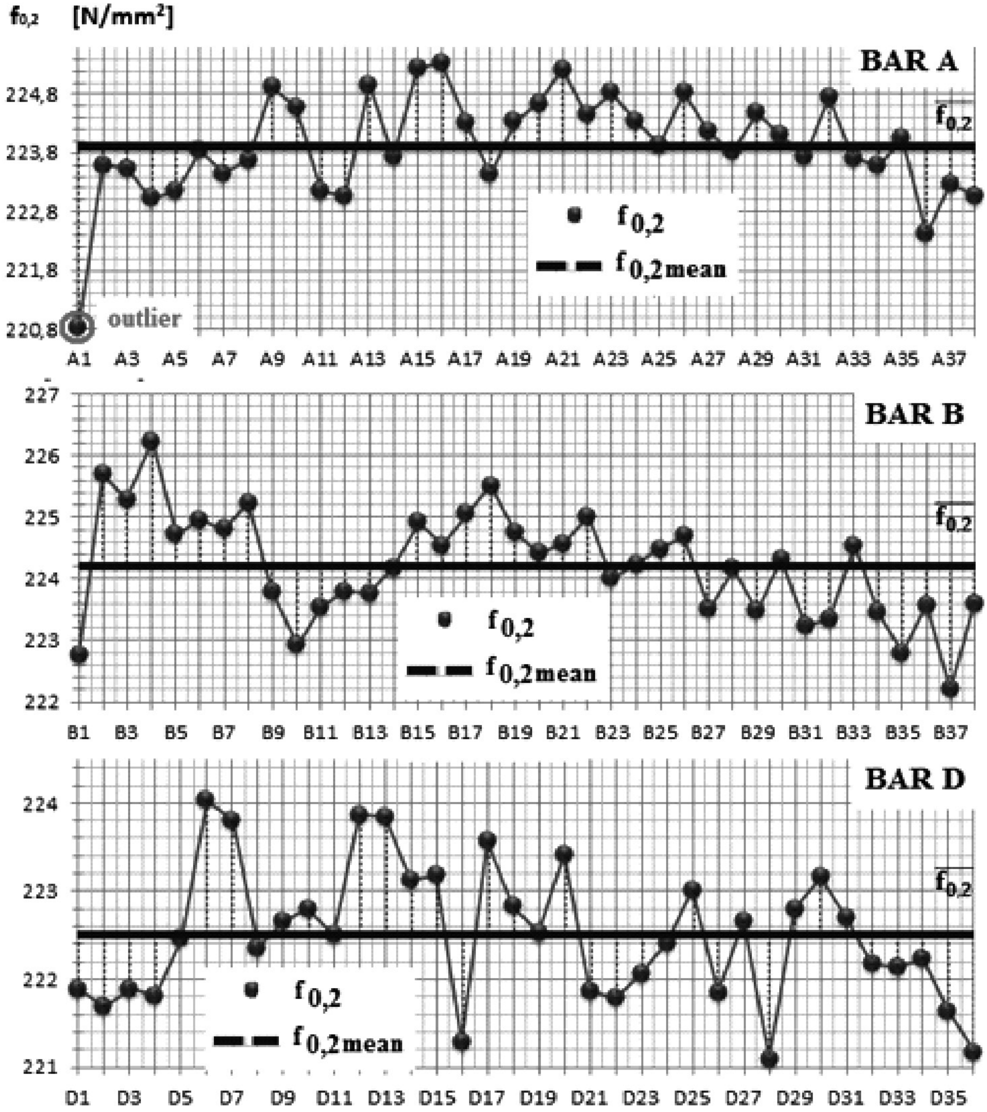

Fig. 4. The sparklines for yield strength – A, B, D bars

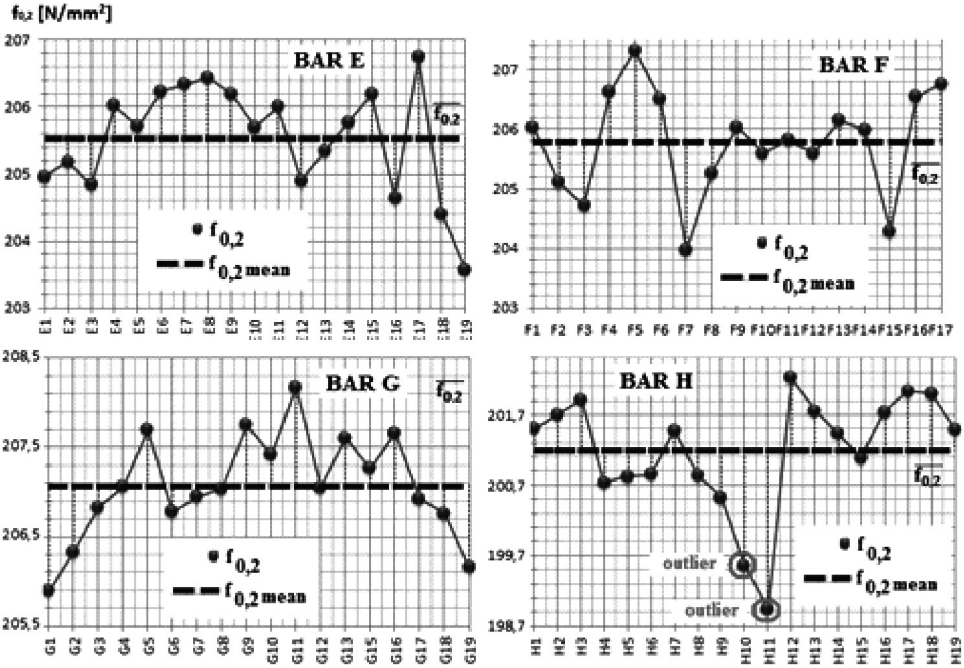

Fig. 5. The sparklines for yield strength – E, F, G, H bars

The graphs (Fig. 6) shows the realizations of ultimate strength f_u for 38 specimens cut from the bar A, 38 from bar B and 36 from the bar D and (Fig. 7) 19 specimens cut from the bar E, 17 from the bar F, 18 from the bar G and 19 from bar H.

The modulus of elasticity was estimated using a computer program (operating testing machine on which the tensile test was carried out) for the stress in the range of from about 25% to 50% of the yield strength. For the calculation of the elastic modulus E was used the linear correlation model. The sparklines estimated in this way, the values of E are shown in figure (Fig. 8) for 38 specimens cut from the bar A, 38 from bar B and 36 from the bar D and 19 specimens cut from the bar E, 17 from the bar F , 18 from the bar G and 19 from bar H.

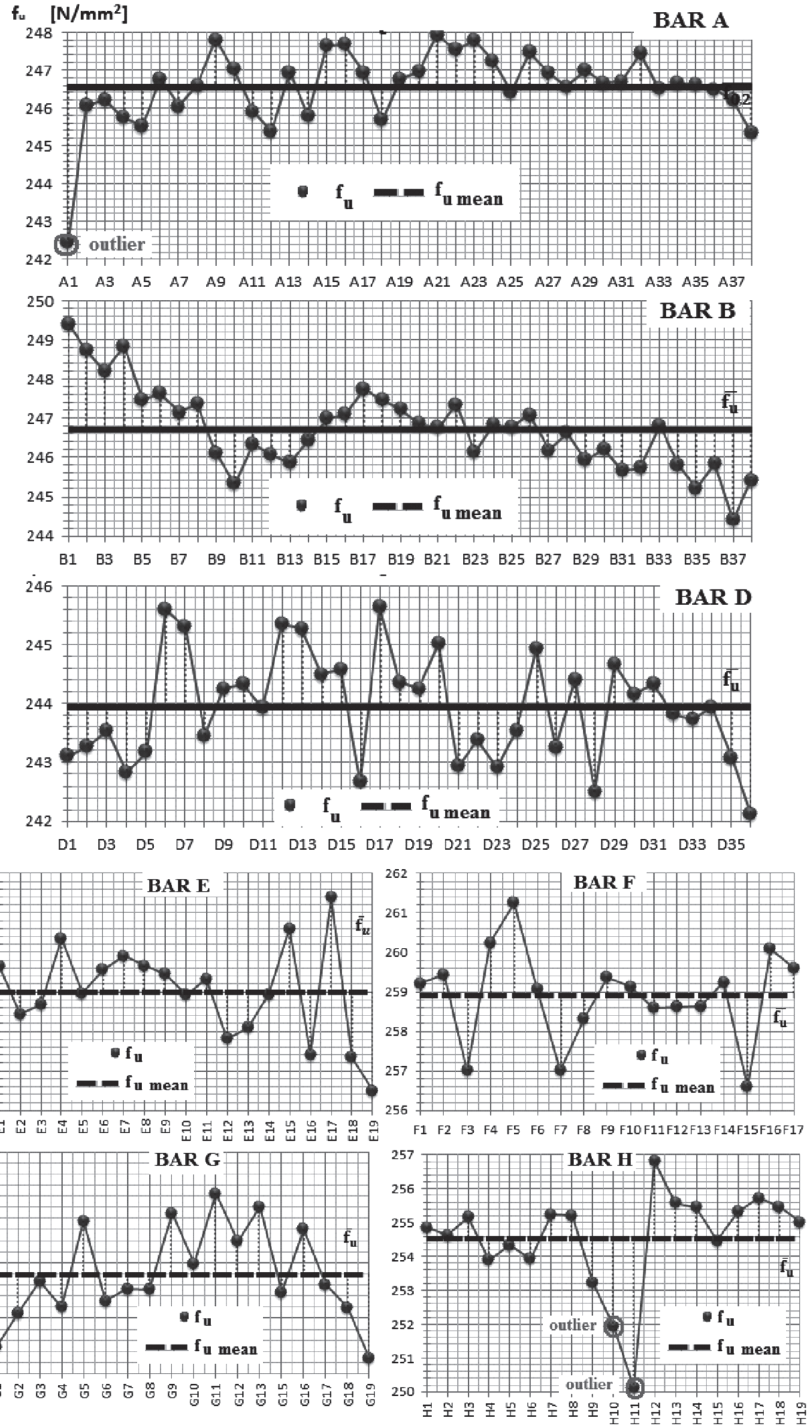

Fig. 6. The sparklines for ultimate strength – E, F, G, H bars

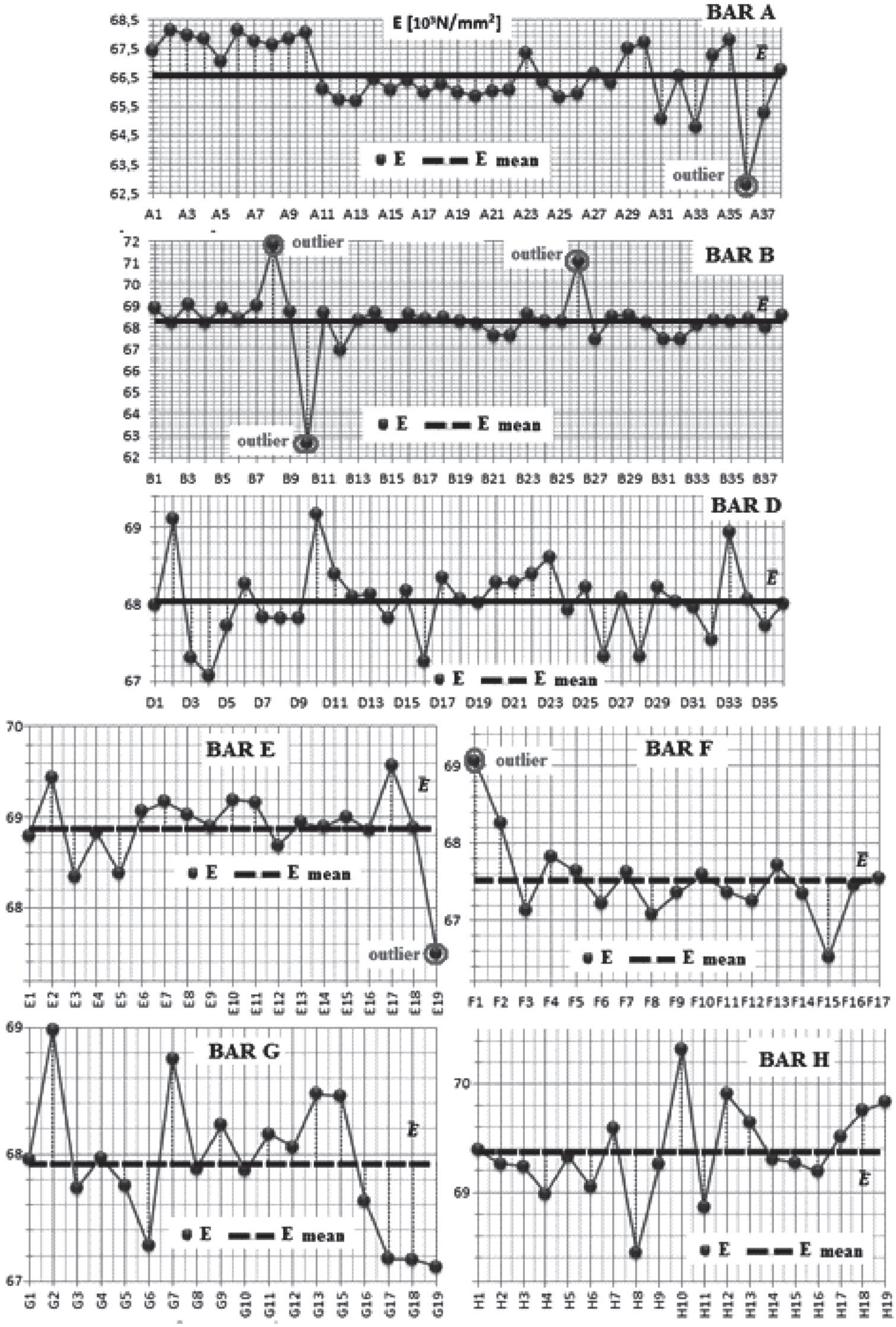

Fig. 7. The sparklines for Young's modulus E – E, F, G, H bars

4. Analysis of test results

The main objective of the study is to estimate the variability of mechanical properties of selected aluminium alloys. Static tensile test gives information about the characteristics of the local minimum at a length of measuring base sample. The first step prior to the analysis of the results obtained to identify and reject outliers in a data set.

4.1. Detection of outliers

In a data set outliers can occur [5, 6], which may be due to [7] approximation error, omission, bias or error thick (mistakes, mistakes in reading and writing performance, improper preparation of the sample or its attachment in the jaws of the testing machine). Typically, these results identified as an extreme value in the results obtained under the same conditions (obtained by the same method in the same laboratory using the same equipment, at short intervals) [8] Results of the study including outliers are characterized by statistical heterogeneity. In order to exclude them from further analysis of the results of appropriate tests statistics. In the literature you can find many methods to verify the results questionable, such as: Q-Dixon [9, 10] or Grubbs test [5]. In Table 2 and Table 3 are presented summaries of the results after the rejection of outliers.

Table 2

Summary of the results after the rejection of outliers for: yield strength $f_{0,2}$, tensile strength f_u and Young's modulus *E* measurements, for bars in alloy EN-AW 6060

Bar	*n*	Mean $\bar{x}$	Variance s^2	Standard deviation *s*	Coeff. of variation *v* [%]
yield strength $f_{0,2}$ [MPa]					
A	37	224.023	0.521	0.721	0.32
B	38	224.215	0.789	0.888	0.40
D	36	222.508	0.612	0.783	0.35
tensile strength f_y [MPa]					
A	37	246.677	0.504	0.710	0.29
B	38	246.720	1.104	1.050	0.43%
D	36	243.951	0.852	0.923	0.38%
Young's modulus E measurements					
A	37	66.692	0.894	0.946	1.42%
B	35	68.293	0.230	0.480	0.70%
D	36	68.037	0.229	0.478	0.70%

Table 3

Summary of the results after the rejection of outliers for: yield strength $f_{0,2}$, tensile strength f_u and Young's modulus *E* measurements, for bars in alloy EN-AW 5754

Bar	*n*	Mean $\bar{x}$	Variance s^2	Standard deviation *s*	Coeff. of variation *v* [%]
yield strength $f_{0,2}$ [MPa]					
E	19	205.535	0.677	0.801	0.39%
F	17	205.786	0.807	0.898	0.44%
G	18	207.076	0.349	0.591	0.29%
H	17	201.421	0.269	0.519	0.26%
tensile strength f_y [MPa]					
E	19	258.002	1.479	1.216	0.38%
F	17	258.909	1.446	1.202	0.46%
G	18	260.785	0.742	0.861	0.33%
H	17	254.958	0.695	0.834	0.33%
Young's modulus E measurements					
E	18	68.951	0.096	0.309	0.45%
F	16	67.432	0.144	0.379	0.56%
G	18	67.924	0.289	0.538	0.79%
H	19	69.376	0.166	0.418	0.60%

4.2. Statistical analysis of the homogeneity of selected mechanical properties

The condition for the applicability of the Analysis of Variance is normality and homogeneity of variance of the variable for all the compared populations [12]. In order to verify the normal distribution of compatibility tests performed Shapiro-Wilk's test and Pearson's Chi-square test. Both tests gave no reason to reject the hypothesis of normal distribution of yield strength, tensile strength and modulus of elasticity for all bars in 5xxx and 6xxx series at the given level of significance $\alpha = 0.05$. The hypothesis of homogeneity of variance was tested using the Bartlett's test. The test results are summarized in Table 4.

Only for 6xxx series (bars A, B and D) must be rejected the hypothesis of equality of variances at the given level of significance. To be able to use the ANOVA procedure must be converted using the so-called stabilization of the variance.

Verification of the hypothesis of equality of means using Analysis of Variance carried out on the assumption that the distribution of the dependent variable results in each group is similar to the normal group compared to a similar size, the individual observations are independent and the variances in the groups are similar. The ANOVA test results are summarized in Table 5.

Table 4

Results of Bartlett's test for bars in alloy EN-AW 6060 and EN-AW 5754

The Bartlett's Test															
Yield strength $f_{0,2}$ [MPa]						Tensile strength f_y [MPa]					Young's modulus E measurements				
Bar	n	s^2	χ^2	$\chi^2_{0,05}$		n	s^2	χ^2	$\chi^2_{0,05}$		n	s^2	χ^2	$\chi^2_{0,05}$	
A	37	0.521				37	0.504				37	0.894			
B	38	0.789	1.60	5.99	+	38	1.104	5.42	5.99	+	35	0.230	23.20	5.99	-
D	36	0.612				36	0.852				36	0.229			
E	19	0.677				19	1.479				18	0.096			
F	17	0.807	6.25	7.82	+	17	1.446	4.00	7,82	+	16	0.144	5.22	7.82	+
G	18	0.349				18	0.742				18	0.289			
H	17	0.269				17	0.695				19	0.166			

Table 5

Results of ANOVA test for bars in alloy EN-AW 6060 and EN-AW 5754

ANOVA											
	k	N	SSb	SSw	MSb	MSw	F	DF k–1	DF N-k	$F_{crit(0,05)}$	H0
Yield strength $f_{0,2}$ [MPa]											
A+B+D	3	111	63.979	69.360	31.989	0.642	49.810	2	108	3.09	–
E+F+G+H	4	71	311.230	35.318	103.74	0.527	196.81	3	67	2.742	–
Tensile strength f_y [MPa]											
A+B+D	3	111	183.688	88.766	91.844	0.822	111.745	2	108	3.09	–
E+F+G+H	4	71	308.27	73.489	102.76	1.097	93.683	3	67	2.742	–
Young's modulus E measurements											
A+B+D	3	108	53.829	48.017	26.914	0.457	58.854	2	105	3.08	–
E+F+G+H	4	71	42.375	11.845	14.125	0.177	79.9	3	67	2.742	–
SS (total sum of squares) = SSb (between) +SSw (within) DF (degree of freedom) MS (mean square)											

The hypothesis H0 was rejected in the Analysis of Variance, so must be carried out further tests (post hoc tests), involved multiple comparisons. After the completion of the group obtained average values, which do not significantly differ. These tests showed that only mean values for pairs of bars A+B and E+F for yield strength are not significantly different.

4.3. Stationarity analysis

The production process can introduce harmonic components with a period longer than the length of the bar, so may be impossible to detections its along the short bars. Statistical tests are performed separately for each bar and the implementation of sparklines, present the general shape of the variation in mechanical properties, could help to exclude potential presence of trend and periodic components. Independence results for specimens can be seen from the graph of autocorrelation function. An example of the stationary process is “white noise” process.

Identification of the trend has influence on the appropriate method of statistical analysis. There are no proven techniques allow for the identification of the components of the trend, however can be seen if it is constantly growing/increasing. In the literature, [13–15] are described stationary statistical analysis through the use of such unit root tests, so it is possible to identify the type of non-stationarity in the data, which can be used to remove any trend in order to bring the data into a stationary process . The pre-existence of a trend, it is possible to detect by visual diagrams and simple regression analysis. Based on the calculated regression coefficients determined the probability test, which compared to the accepted level of significance.

In the analysis of one-dimensional random function uses two basic features that characterize this variability: the autocorrelation function or function autocovariance. According to [16] field autocorrelation function should be limited up to one third the length of the measured execution. The function was calculated for the step Δx = 15 cm corresponding to the length of the specimen, and the maximum delay equal to half the length of the bar. In the case where the test process is stationary, autocorrelation function values should be close to zero, which means that it is uncorrelated sequence of random variables with a fixed variance and null mean value. For this purpose, the analysis of autocorrelation functions performed for centralized.

To verify the hypothesis of no first-order autocorrelation of the random component is used Durbin–Watson’s test, under conditions of normal distribution of the random component. Durbin – Watson test indicates a positive first-order autocorrelation yield for bars: A, B, E and F. For the rod D, and G, H, there are no autocorrelation. The figure shows the graph autocorrelation function (Fig. 8) for the yield of bars A, B and D.

Statistical significance of further investigating the correlation coefficient is the Ljung-Box statistics form. Autocorrelation coefficients are individually significant for delays $R = 1$ and $R = 18$ for the bar A. For the bar B, first few rows of autocorrelation is important – there is a trend , confirms that visual analysis of scatter plot points of the empirical and the course of the auto-regression function . There are also random fluctuations and no periodic fluctuations. In the case of the bar D, the autocorrelation is not observed.

Fourier analysis (spectral) [17] is used to study the harmonic structure of the time series (random function). The purpose of this analysis is to determine the number of the power spectrum versus frequency or period (Fig. 9).

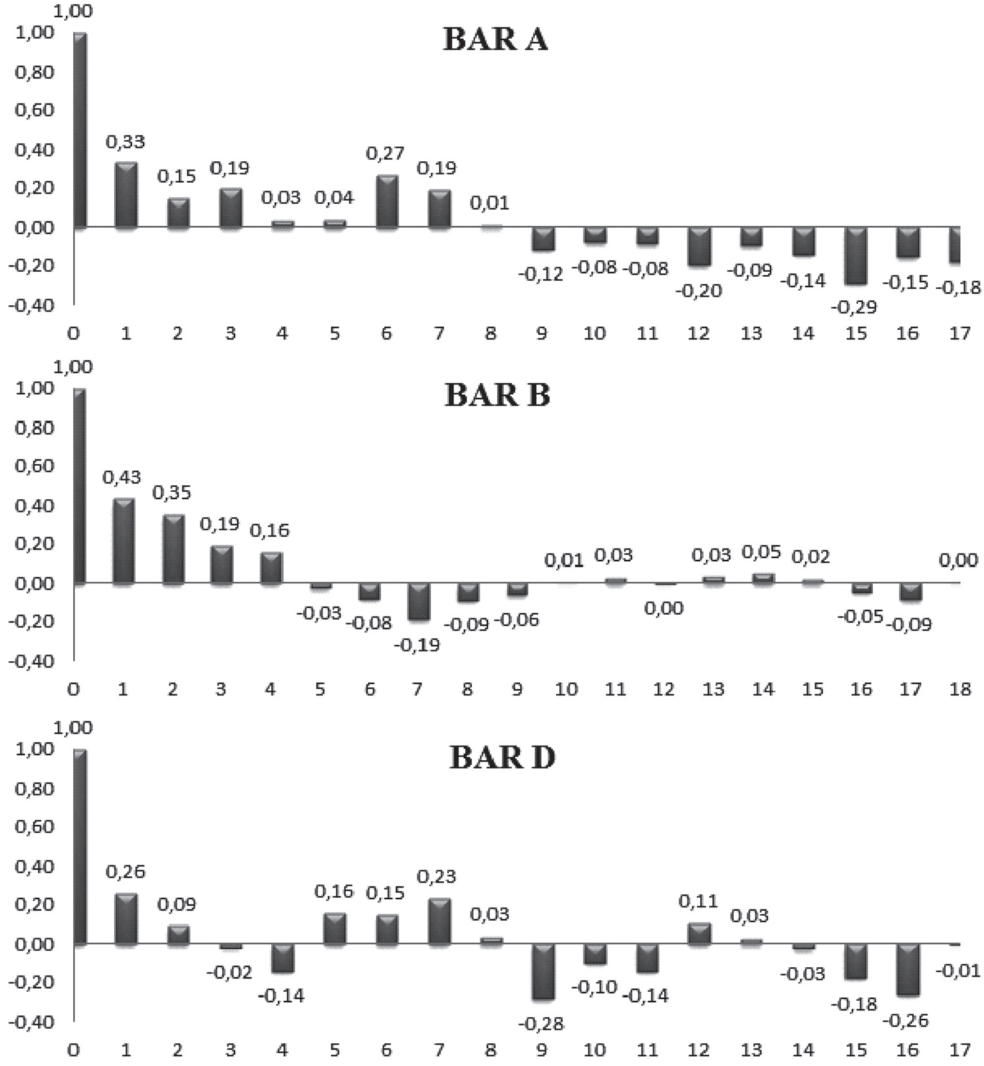

Fig. 8. Autocorrelation function graph for the yield strength

Decomposition of time series, looking like random noise, allows discovering some periodic cycles of different lengths. Prior to spectral analysis, potential trends must be removed, and the average should be subtracted to receive stationary process.

The process can be considered as white noise if its components are normally distributed and the value of the periodogram (Fig. 10) will have an exponential distribution. For this purpose, compliance testing is carried out, f.eg. Kolmogorov – Smirnov's test [18].

If the harmonic period is less than the length of realization (bar), the periodic component should be distinguished by the occurrence the peak in a function of spectral density (Fig. 9).

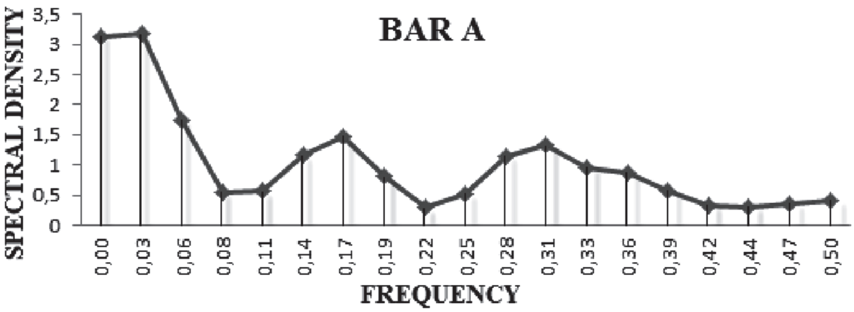

Fig. 9. Spectral density graph for yield strength

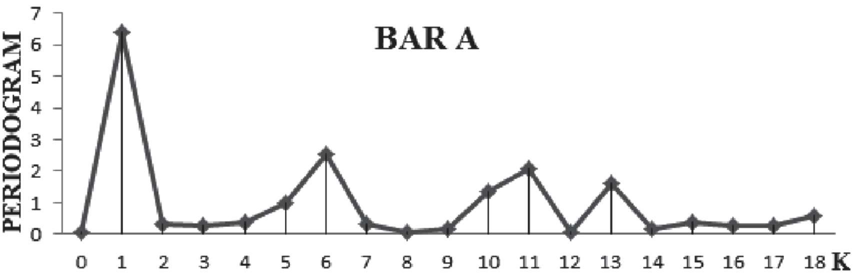

Fig. 10. Periodogram for yield strength

5. Conclusions

The study of the mechanical properties of selected aluminum alloys, are shown graphically in the following figures (Fig. 4–7) as realizations of random function corresponding to a hypothetical general population bars meet the requirements of homogeneity. Summaries of the results for: Young's modulus E, yield strength $f_{0,2}$, tensile strength f_u for bars in alloy EN-AW 6060 and EN-AW 5754 are shown in the Table 2 and Table 3. An initial inspection of the implementation of graphs, simple regression analysis and autocorrelation analysis helped to identify the structure of the outputs as a stochastic process. After elimination of possible trends and bringing the process into a stationary spectral analysis was carried out to demonstrate the absence of significant periodic signals nonstationarity on the length of the rods. Therefore, to describe the variability of strength for both parties of bars from 5xxx and 6xxx series aluminum alloys can be assumed the stationary model similar to the white gaussian noise.

The obtained results show a slight heterogeneity of mechanical stochastic local features. In particular, the coefficients of variation of the yield strength and ultimate strength are less than 1% (see variance according to Table 2 and Table 3). For comparison, for flats made of steel *St3S* [19], where he obtained values of the coefficients of variation v = 2.4–2.8%. Differences in the size and the measurement of displacement in the distribution of the values of the outputs of the coefficient of variation described aluminum and steel products have also been observed in [20]. Based on the results of this analysis can be pre-concluded that the stochastic variance [21–23], which is one of the three components of variance (beside statistic variance and probability variance) is so small that there is no significant effect on the coefficient of variation γ_M=1,1 proposed by Eurocode 9 [24]. It should also be noted that the strength test was carried out at that time, the strength of the older generation machines, which were not equipped with electronic measurement recording systems, individual embodiments are therefore less accurate than contemporary recorded.

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