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EFFECT OF SOME FACTORS ON SECOND ORDER EFFECTS IN REINFORCED CONCRETE COLUMNS

WPŁYW WYBRANYCH CZYNNIKÓW NA EFEKTY II RZĘDU W SŁUPACH ŻELBETOWYCH

A b s t r a c t

This paper presents the results of an analysis of reinforced concrete columns in which there are differences in the value of the second order effects obtained from the method of nominal stiffness (MNS) and the method of nominal curvature (MNC) based on Eurocode 2. Some of the factors such as cracking, ratio of reinforcement, cross-section of a column, and relation of an axial force N_{Ed} to a design axial resistance of section $N_{Rd,}$ were analysed. It was shown that the choice of the method for calculating the second order effect is crucial and can impact the design of columns significantly.

Keywords: concrete columns, second order effects, method based on nominal stiffness, method based on nominal curvature, effective length, cracking

S t r e s z c z e n i e

W artykule przedstawiono wyniki prac i własne analizy dla żelbetowych słupów, w których widoczne są różnice wartości efektów II rzędu, określonych metodami nominalnej sztywności i nominalnej krzywizny wg EC2. Analizując wpływ zarysowania elementów, stopnia zbrojenia, przekroju słupa i stosunku siły podłużnej do siły krytycznej na efekty II rzędu, pokazano, że dobór metody obliczeń efektów II rzędu może znacząco wpływać na wyniki wymiarowania słupów.

Słowa kluczowe: słupy żelbetowe, efekty drugiego rzędu, metoda nominalnej sztywności, metoda nominalnej krzywizny, długość wyboczeniowa, zarysowanie

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1. Introduction

In the analysis of slender reinforced concrete columns subjected to longitudinal force and bending moment, the influence of deformations of the structure on internal forces should be considered. Second order effects are these additional effects, bending moments or eccentricities.

Depending on the kind of construction, with the aim of considering these effects, a global analysis is made – whole construction calculations (which should also take into account the influence of cracking, creep and non-linearity of material properties). The other method is to use a local analysis of isolated members (columns).

The following article concentrates on columns, which can be treated as isolated. The procedure in this case can be simplified to two basic steps:

- static analysis of the whole construction, based on the rule of stiffness (first order analysis),
- checking the slenderness of each column and comparing it with the proper limit values of slenderness. If the slenderness exceeds the limit value, additional moments (eccentricities) caused by deformations of members should be calculated (second order analysis).

There is an increase of bending moments due to second order effects in compressed concrete members. The final value of M_{Ed} moment, which is taken to calculations, consists of:

$$M_{Ed} = M_{0Ed} + M_2 \tag{1}$$

where:

M_{0Ed} – 1st order moment, including the effect of imperfections,
M_2 – nominal 2nd order moment.

In Eurocode 2 [17] three methods used for second order effects analysis are indicated:

- general method – based on non-linear second order analysis,
- simplified method – based on nominal stiffness (MNS),
- simplified method – based on nominal curvature (MNC).

The general method of calculating load-bearing capacity for columns considering second order effects has not been specifically defined in Eurocode 2. It can be assumed that this name is used to describe approaches in which the deformation of the column is not assumed at the beginning. Therefore, it is determined by analysis of subsequent cross-sections at the column length. There is a necessity in this approach to use computer programmes, like in the method described and verified experimentally by M.E. Kamińska and A. Czkwianianc in their paper [1].

Papers [2, 8] inform that methods of analysis of columns apply only to isolated members of constant cross-section and reinforcement, which are subjected to loading only at their ends.

In p. 5.8.5 of Eurocode 2 [17] there is a statement that methods based on nominal stiffness as well as the one based on nominal curvature can be used for isolated members. In 5.8.8.1 (1) of [17] it is said that the nominal curvature method is "primarily suitable for isolated members with constant normal force and a defined effective length l_0". Its usability is restricted by p. 5.8.8.3 of Eurocode 2 to members of constant, symmetrical cross-section with symmetrical reinforcement.

In the Polish National Annex to Eurocode 2, any of the simplified methods (MNS or MNC) were designated as binding ones. The lack of criteria of choosing the proper method is an important problem due to the fact that the results of calculations of second order effects with two methods mentioned above can be significantly different in lots of cases.

The example described in B. Westerberg paper [14] can be an illustration for this statement. The paper consists of the analysis of the 6.0 m long cantilever column with a cross-section of 0.8 × 0.6 m, subjected on its top side to a transverse force H_{Ed} = 200 kN in the plane of higher stiffness. In the plane of lower stiffness, the column was loaded with a vertical force N_{Ed} = 3000 kN with an eccentricity of e = 0.2 m. Among the material assumptions, the design value of concrete compressive strength was taken as f_{cd} = 20 MPa and design yield strength of reinforcement as f_{yd} = 435 MPa. Second order moments have been calculated with two methods – MNS and MNC (independently in both planes of the column) and the reinforcement calculated for those cases was compared. In the plane of higher stiffness, moments obtained from second order effects calculated by MNS are 1.4 times higher than those from MNC, whereas for the plane of lower stiffness, the factor of the difference increases to 2.4 (that is: for MNS: 1265 kNm, for MNC: 525 kNm).

The relation of second order moments (calculated in the plane of lower stiffness with two methods) to the value of $\omega = A_s f_{yd}/(A_c f_{cd})$ and in comparison to M_{Rd} is shown in Fig. 1 [14].

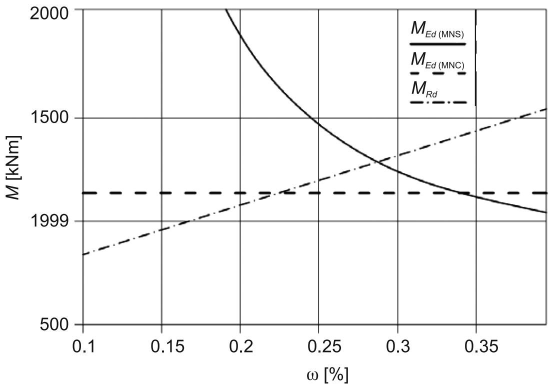


Fig. 1. The relation of second order moments (MNS, MNC) and M_{Rd} to the value of reinforcement intensity ω (for y – axial bending) [14]

The difference between second order effects, calculated with MNS and MNC, can also be noticed on the graphs presented by M.E. Kamińska in paper [5]. Graphical results of calculations of second order eccentricities for columns are presented for various cases: with slenderness l_0/h = 10, 20, 30 i 40, with reinforcement ratio $\rho_{L1} = \rho_{L2}$ = 0,0164 and $\rho_{L1} = \rho_{L2}$ = 0.0027, for strength classes for concrete B20 i B60 and for RB500W for steel. Calculations were performed based on EC2 from the year 2002 as well as PN-02 [16] and were compared with the results from the analytical method.

It can be noticed that the differences between second order effects calculated with MNS and MNC are significant, especially for columns made of lower concrete grades (B20), lower reinforcement ratio ($\rho_{L1} = \rho_{L2}$ = 0.0027) and higher slenderness (l_0/h = 30; 40) with e_0/h = 0.1. The values of second order effects from MNS are then higher than those from MNC; for example for l_0/h = 30 the eccentricity e_2/h for MNS is two times bigger than from MNC.

It is necessary to underline that the values of eccentricities from MNC shown in the graphs presented in the paper [5], are in lots of cases similar to the results obtained from the analytical method. On the other hand, the results from other methods (including MNS) differentiate more, depending on the column slenderness.

K. Koziński, in the paper [7], compared the second order effects obtained from the experimental studies with the results from analytical calculations using various methods: MNS, MNC according to EC2 [17], ACI method [15] and PN-02 method [16]. The results for 16 columns subjected to compression with eccentricities on both axes were analysed. The effective length of the columns was equal to 2.81 m, the cross-sections: 150 × 150 mm; 150 × 300 mm; 150 × 450 mm; 150 × 600 mm. The studies were conducted by subjecting the columns to the assumed loads with eccentricities equal to 50 mm and 150 mm and with the plane deviation angles equal to 22,5° and 45°. The reinforcement ratio of the analysed columns was constant with the value of 2.74%, steel used was RB500W. The columns were made of high concrete grades – the compression strength after 28 days was $f_{cm,cube}$ = 122.8 MPa.

K. Koziński claimed that, on the basis of calculations and the experimental results, the differences between the values obtained from various methods are significant and the second order effects from MNS (similar to those from PN-02) are several times over-estimated. The results of calculations according to ACI were the closest to the experimental data of transverse shifts of the tested columns. The MNC results were less accurate, however, still not so different from the measured values.

The comparing analysis of the influence of the chosen factors on the second order effects in RC columns calculated with the method of nominal stiffness and the method of nominal curvature are presented in the second part of the article. In each case, the action of N_{Ed} and M_{Ed} (strong – axis bending) was considered.

2. Simplified methods of calculating second order effects

2.1. Method of nominal stiffness (MNS)

Method of nominal stiffness is based on the critical force due to the buckling calculated for the nominal stiffness of the analysed member. It is advisable that the material non-linearity, creep and cracking, which have an impact on the behaviour of the structure members, are taken into consideration. The design moment in the members subjected to the bending moment and an axial force which includes the effect of the first and second order effects, can be shown as a bending moment increased by the factor described below:

$$M_{Ed} = M_{0Ed} + M_2 = M_{0Ed} + M_{0Ed} \cdot \frac{\beta}{\frac{N_B}{N_{Ed}} - 1} = M_{0Ed} \cdot \left(1 + \frac{\beta}{\frac{N_B}{N_{Ed}} - 1} \right) \tag{2}$$

where:

M_{0Ed} – 1st order moment, including the effect of imperfections,
M_2 – nominal 2nd order moment,
N_B – buckling load based on nominal stiffness,

N_{Ed} – design value of axial load,
β – factor which depends on distribution of the 1st and 2nd order moments.

2.2. Method of nominal curvature (MNC)

The method of nominal curvature allows for the calculation of the second order moment based on the assumed curvature distribution (which responds to the first order moment increased by the second order effects) on the length of the member. The distribution of the total curvature can be either parabolic or sinusoidal.

The value of the II order moment can be calculated as:

$$M_2 = N_{Ed} \cdot e_2 \tag{3}$$

where:

N_{Ed} – design value of axial load,
e_2 – deflection calculated by taking into account such parameters as creep, intensify of the reinforcement and also distribution of the reinforcement over the height of the cross-section.

$$e_2 = \frac{1}{r} \cdot \frac{l_0^2}{c} \tag{4}$$

where:

c – factor depending on the curvature distribution,
l_0 – effective length,
$1/r$ – curvature.

Considering the formula (4), the determination of the l_0 value is particularly important because it influences on the e_2 eccentricity with the second power.

In the aim of determining the curvature of members with a constant, symmetrical cross-section, Eurocode 2 allows the equation:

$$\frac{1}{r} = K_r \cdot K_\varphi \cdot \frac{1}{r_0} \tag{5}$$

where:

$1/r_0$ – basic curvature:

$$\frac{1}{r_0} = \frac{f_{yd}}{0.45 \cdot E_s \cdot d} \tag{6}$$

f_{yd} – design yield strength of reinforcement steel,
E_s – design value of modulus of elasticity of reinforcement steel,
d – effective depth of the cross-section if the reinforcement is located on both sides of the cross-section or substitute effective depth if part of the reinforcement is placed along the cross-section depth, parallel to the bending plane,
K_φ – factor for taking account of creep,

$$K_\varphi = 1 + \beta_\lambda \cdot \varphi_{ef} \geq 1,0 \tag{7}$$

K_r – correction factor depending on axial load,

β_λ – factor for taking account of characteristic compressive cylinder strength of concrete and slenderness ratio.

A β_λ coefficient (β_λ symbol was used to differentiate between β coefficient in MNS equations) is determined as:

$$K_\varphi = 1 + \beta_\lambda \cdot \varphi_{ef} \geq 1,0 \tag{8}$$

where:

f_{ck} – characteristic compressive cylinder strength of concrete after 28 days,

λ – slenderness ratio.

A K_r coefficient, which allows for the decreasing of the curvature of the element for higher axial forces values is calculated as:

$$K_r = \frac{n_u - n}{n_u - n_{bal}} \leq 1.0 \tag{9}$$

where:

n_u – $1 + \omega$,

ω – intensity of reinforcement, $A_s f_{yd}/(A_c f_{cd})$,

n – relative axial force, $n = N_{Ed}/(A_c f_{cd})$,

n_{bal} – relative axial force n in the case in which the the maximum limit value of a moment is achieved. According to the EC2 the value $n_{bal} = 0.4$ can be established,

A_c – area of concrete cross section.

The description of how the K_r coefficient depends on the n_{bal} is included in p. 4.3 of this paper.

3. Effective length of columns as a function of their flexibility in nodes

According to the EC2 [17], the effective length l_0 is determined to consider the shape of the deflection curve (caused by buckling). It is the length of a column with joints on both ends, subjected to constant axial force, with the same cross-section and buckling load as the analysed member. Eurocode 2 includes examples of different buckling modes and the corresponding effective lengths for isolated members with constant cross-sections.

For frames with the regular mesh of columns and beams, Eurocode gives relationship defining the l_0 lengths for isolated members. The general equation is:

$$l_0 = \beta \cdot l \tag{10}$$

where:

β – factor of buckling,

l – clear height of compression member between end restraints.

The β coefficient is calculated according to Eurocode equations numbered (11) and (12), depending whether the structure is braced or not.

For braced members:

$$\beta = 0.5 \cdot \sqrt{\left(1 + \frac{k_1}{0.45 + k_1}\right) \cdot \left(1 + \frac{k_2}{0.45 + k_1}\right)} \tag{11}$$

For unbraced members:

$$\beta = \max\begin{cases} \sqrt{\left(1+10\cdot\dfrac{k_1\cdot k_2}{k_1+k_2}\right)} \\ \left(1+\dfrac{k_1}{1+k_1}\right)\cdot\left(1+\dfrac{k_2}{1+k_2}\right) \end{cases} \tag{12}$$

where:

k_1, k_2 – relative flexibilities of rotational restraints at ends 1 and 2.

They are calculated using an equation:

$$k = \frac{\theta}{M}\cdot\frac{EJ}{l} \tag{13}$$

where:

θ – rotation of restraining members for bending moment M,

EJ – bending stiffness of compression member.

This relationship after simple transformation can be explained as follows:

$$k = \frac{\alpha_{rs}\left(\dfrac{EJ_a}{l_a}+\dfrac{EJ_b}{l_b}\right)}{k_i\cdot\alpha_{rr}\cdot\sum\dfrac{EJ_r}{l_{eff}}} \tag{14}$$

where:

$\sum\dfrac{EJ_r}{l_{eff}}$ – sum of relative stiffness of beams (bracing members) in a node in an analysed plane,

$\dfrac{EJ_a}{l_a}, \dfrac{EJ_b}{l_b}$ – relative stiffness of column above and below the node,

k_i – coefficient equals 3.0 for pin-ended and 4.0 for restrain,

α_{rs}, α_{rr} – reduction coefficients considering stiffness decrease of beams or columns due to cracking, described below.

If there is a possibility that the compressed member which adheres to the analysed element has an impact on the rotation caused by buckling, it should be included in the calculation of the k coefficient. Therefore symbols a and b have been added to the above mentioned equation – they stand for compressed members (columns) above and below the analysed node.

The influence of cracking can be considered using various approaches with different precision and labour demands, as described in paper [13] for example. The most accurate method is based on determining the stiffness distribution which responds to the moment caused by subjected loads. It is evaluated in a finite number of nodes on the beam length, and the rotation angle caused by the moment in the node is calculated later. However, this method demands sophisticated numerical calculations which can be completed only with the use of computer programmes. A simpler approach is based on the idea of calculating the lowest cross-section stiffness on the length of the beam after its cracking. The most simplified method is to use the decreasing coefficients for stiffness of the members. Respectively, for the full stiffness the coefficients equal to 1.0, while the highest decrease of stiffness is described with the lowest values of these coefficients.

From the comparison of stiffness reducing coefficients presented below, the conclusion is that in many cases, the ratio α_{rc}/α_{rb} equals 0.50.

The approach based on decreasing stiffness is used for example in the ACI-318 [15], where the reduction of stiffness is assumed as follows:

- columns $\alpha_{rc} = 0.70$,
- beams $\alpha_{rb} = 0.35$.

The same values of reduction factors due to cracking are suggested by W. Starosolski in paper [11].

J. MacGregor [9, 8] claims that columns before failure are usually not as cracked as beams, therefore he determines the reduction factors as:

- columns $\alpha_{rc} = 1.00$ without any reduction,
- beams $\alpha_{rb} = 0.50$.

According to another paper by J. MacGregor and S. Hage [10, 8], concrete member stiffness can be reduced by factors:

- columns $\alpha_{rc} = 0.80$,
- beams $\alpha_{rb} = 0.40$.

It seems interesting to analyse how the change of stiffness of beams and columns caused by cracking influences the stiffness of the node or the β coefficient and therefore the second order effects [4].

4. Parametrical analysis

The following part presents a comparative analysis of the influence of certain factors on second order effects in the reinforced concrete columns in braced and unbraced structures. The analysis takes into consideration the following factors: the stiffness of cracked elements (both beam and column), ratio of the reinforcement, area of the cross-section of the column and the relation between design axial force N_{Ed} and design axial resistance of section N_{Rd}.

The subject of this analysis is an isolated column of a frame structure (of typical dimensions for these structures). Two columns with different slenderness (element I and II) in braced structure and one column (element I) in unbraced structure were analysed, assuming the same material characteristics: concrete grade of C30/37 and steel RB500W.

Dimensions, static scheme and support conditions of columns were assumed as is shown on the Fig. 2. In the case of the unbraced structure, the horizontal displacement of beams was allowed (node 2). The dimensions of the cross-section of beams (30 × 60 cm) and the distance between the faces of the supports (5.95 m and 3.00 m) were established. The column I has dimensions of the cross-section: 40 × 50 cm and its height (between the faces of the supports) equals 4.10 m. The respectively dimensions of the column II are: 30 × 40 cm (cross – section) and 6.50 (height).

For comparative purpose, in those analyses in which the following parameters were constant, it was assumed that the results of static analysis did not include the influence of the deformations of structure (equal for restrained and unrestrained structure). The bending moment in the top node 150 kNm and in the bottom one 60 kNm, an axial force 1450 kN and the reinforcement 4φ16 were established.

The support flexibility of the node /1/ $k_1 = 0.1$ (pin – ended) was assumed. It is because of the fact that in reality, the realization of fully fixed support for which the adequate value is k = 0 (theoretic limit for fully fixed support), is really rare [17]. The support flexibility of the node /2/ was calculated from the relation (11) or (12). The effective creep coefficient φ_{ef} = 1.32 in the braced structures and φ_{ef} = 1,24 in the unbraced structure were established. The value c_0 = 8 was taken into the MNS method for both braced and unbraced structures. In the first case, due to the constant equivalent moment and in the later, due to the fact that this value generates the highest second order effects.

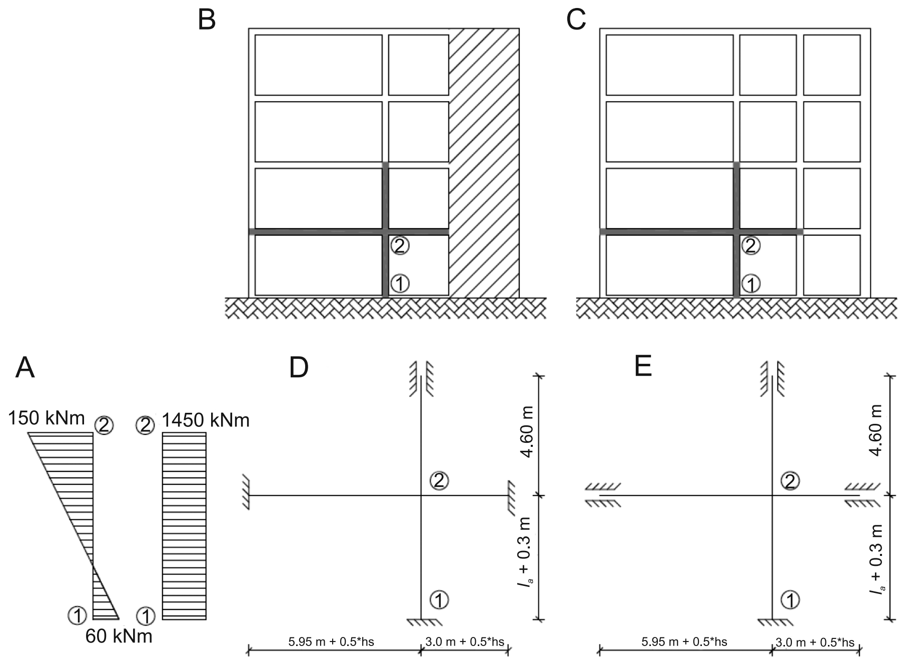


Fig. 2. Forces in the analysed column 1–2 (A) and the static scheme of the part of the braced structure (B, D) and unbraced structure (C, E)

4.1. The influence of cracking on values of the second order effects

Changes in stiffness of the cracked elements and its influence on the support flexibility /2/ and β (11), (12) coefficient and consequently on the size of the second order effects were analysed.

In the analysis, four combinations of stiffness reducing coefficients were taken into consideration, for both braced and unbraced structures:

- case 1 – uncracked beam and column,
- case 2 – uncracked beam and cracked column,
- case 3 – cracked beam and uncracked column,
- case 4 – both beam and column cracked.

Values of the second order effects were calculated according to Eurocode 2 [17] using MNS and MNC. Results of the analysis are presented in Tables 1 and 2 and in graphs 3 and 4 for braced and unbraced structures, respectively.

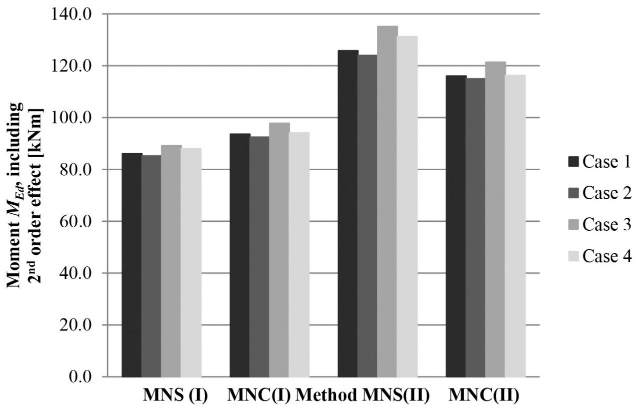
4.1.1. Analysis of a column in a braced structure

In the isolated member of the braced structure, moments with the second order effects for each case (calculated using both methods) are similar. They differ from 3 to 5 percent for column I and from 6 to 9 percent for column II. Case 3 (cracked beam and uncracked column) generates the largest moments with the second order effects. In cases 2 and 4, when the column was also assumed as cracked, these moments with the second order effects decrease. It can be concluded that taking into consideration a possibility of cracking of beams or columns, does not have a significant influence on the results. It is not necessary to evaluate the decrease of the columns stiffness which is caused by cracking, because such assumption increases the safety reserve. Due to the fact that differences of the results for cracked and uncracked beams (cases 1 and 3) are not significant, the stiffness of the beam could be set at a level of 40–50 percent and furthermore, the exact calculations of the stiffness can be omitted.

Table 1

Moments with the second order effects in relation to cracking of members in braced structures for columns I and II

nb.	factor of stiffness		effective length l_0 [m]		equivalent moment [kNm]		moment including 2nd order effect MNS [kNm]		moment including 2nd order effect MNC [kNm]	
	beam	column	I	II	I	II	I	II	I	II
1	1.00	1.00	2.53	3.73	75.17	79.51	86.08	125.83	93.65	116.07
2	1.00	0.70	2.46	3.67	74.93	79.32	85.24	124.02	92.49	114.97
3	**0.35**	**1.00**	**2.76**	**3.98**	**76.02**	**80.41**	**89.15**	**135.16**	**97.80**	**121.37**
4	0.35	0.70	2.68	3.87	75.72	80.04	88.05	131.22	94.06	116.27



Rys. 3. Results of the analysis of the influence of members' cracking on the second order effects

In the analysis of the braced construction, some inconsequences can be observed and these result from the statements in Eurocode 2. The value of limit slenderness λ_{lim} is 3–4 times bigger than the slenderness of member I and 1.5 times bigger than the slenderness of member II. This means that according to EC2, the second order effects should not be taken into consideration. Designers are allowed to not check the second order effects by comparing the slenderness with the limit value and calculating the concrete column as a thick one. However, the values of the second order effects are greater than 10% of the first order moment (equivalent moment). For element I ($\lambda = 17\div19$) the second order effects are 13–17 percent of the equivalent moment for MNS and 23–28 percent for MNC. For the element II ($\lambda = 32\div35$) the ratio equals 56–63 percent for MNS and 45–50 percent for MNC.

4.1.2. Analysis of a column in an unbraced structure

In the analysis of the isolated member of the unbraced structure, the calculated moments, which include second order effects, differ significantly, as shown on Fig. 4, for cases 1–4. In each case, higher values of the effects are obtained from the MNS. In both methods, the calculated moments are higher when considering cracking in the beam only (case 3). The difference between the final moment in the uncracked structure (case 1) and in the structure with the cracked beam (case 3) is more than 20% for MNS and about 8% for MNC.

A conclusion should be made that considering cracking of the beam in unbraced structures is very important. However, it is not necessary to take into account cracking of a column for the same reason as in the braced structures. To avoid over-reinforcement of the column, the best option is to calculate the exact stiffness of the beam. There is also a possibility of assuming the stiffness of the cracked beam as retaining 45–50 percent of the stiffness of the uncracked member. However, the influence of the beams' cracking on their stiffness and a flexibility of nodes and therefore on the effective length of columns, should not be omitted.

Table 2

Values of moments with the second order effects in relation to cracking of elements in unbraced structures

nb.	factor of stiffness		effective length l_0 [m]	equivalent moment [kNm]	moment including 2nd order effect MNS [kNm]	moment including 2nd order effect MNC [kNm]
	beam	column				
1	1.00	1.00	5.26	169.07	286.42	239.06
2	1.00	0.70	5.12	168.58	278.21	235.39
3	**0.35**	**1.00**	**6.01**	**171.79**	**340.62**	**259.81**
4	0.35	0.70	5.67	170.56	313.97	239.15

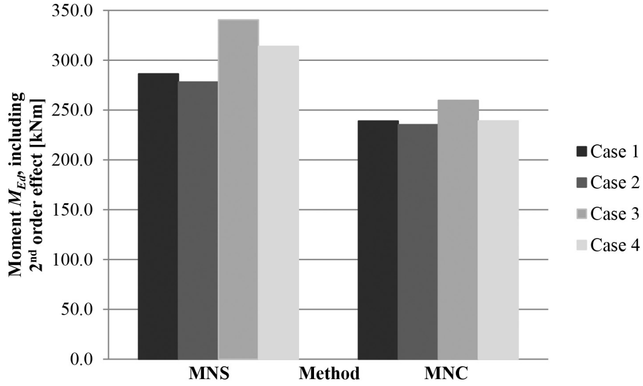


Fig. 4. Results of the analysis of the influence of members' cracking on the second order effects

The differences between the results from MNS and MNC can be easily observed. The values of the moments with the second order effects from MNC and MNS are respectively 70–100 and 40–50 percent greater than the first order moment in columns.

In the analysis of the braced structure, the second order effects are similar for both methods, whereas for the unbraced construction, the second order moments from MNS are 30% higher than those from MNC. According to this remark, the proper choice of method for evaluating the second order effects in an unbraced structure has a significant effect on the design of the column.

4.2. Influence of reinforcement ratio and cross-section of a column on the second order effects

The following analysis presents the influence of the reinforcement ratio in a column in braced construction on the second order effects. The static scheme and support conditions in the column were assumed as on Fig. 2: the bending moment 150 kNm in the top node and 60 kNm in the bottom one and an axial force equals 1450 kN. Material characteristics: concrete grade C30/37 and steel RB500W. The cross-section of the beam is 30 × 60 cm and the distances between the faces of its supports are 5.95 m and 3.00 m. Support flexibility of the node /1/ k_1= 0.1 and of the node /2/ was calculated from the relation (11). Effective creep coefficient φ_{ef} = 1.32 and c_0 = 8.

The only change is the cross-section dimensions, which are equal to 30 × 40 cm, 40 × 40 cm and 40 × 50 cm, respectively. The analysis is conducted assuming the stiffness reduction factor for a beam as α_{rb} = 0.35 and full stiffness of the column. The reinforcement ratio range is from 0.2% up to 0.99% with a step 0.1%.

Calculations were made with both methods, MNC and MNS. Results are presented in Table 3 and Fig. 5.

Results from both methods differ distinctly. In Fig. 5, they are presented in relation to the equivalent moment, which is a constant value. In the analysed cases, differences between moments with the second order effects calculated with both methods reach 50%. Moments from MNS decrease with the increasing reinforcement ratio and the largest decrease happens in the column with the smallest cross-section. The reinforcement ratio does not influence values of the second order effects obtained from MNC. The moment is almost equal, small differences in the graph are caused only by changes of the reinforcement diameter.

The largest second order effect from MNS is for the column with the smallest cross-section, whereas it is entirely the opposite in MNC – the highest second order effects are for the largest column.

The other conclusion is that differences between columns of cross-sections of 30 × 40 cm and 35 × 45 cm are higher than the differences between 35 × 45 cm and 40 × 50 cm. The reason for such a situation is a K_r coefficient, which according to Eurocode 2 is limited to the value of 1.0. The column with the smallest cross-section has a coefficient $K_r = 0.74$, the middle column $K_r = 0.95$ and for the largest column, the calculated coefficient was higher than 1,0, therefore it was assumed as 1.0 – according to the equation (9) of Eurocode 2 and Fig. 6. That is why the difference between moment values for middle and the largest column is slight.

Table 3

Moments with the second order effects values [kNm] determined with MNC and MNS in dependence of the reinforcement ratio and column cross-section (in braced construction)

reinforcement ratio [%]	dimensions of the column [cm]					
	40 × 50		35 × 45		30 × 40	
	Moment M_{Ed}. including 2nd order effect [kNm]					
	MNS	MNC	MNS	MNC	MNS	MNC
0.20	99.10	98.09	108.23	97.47	123.13	92.96
0.30	92.86	98.15	99.81	97.56	111.64	93.20
0.40	89.32	98.20	94.91	97.64	104.45	93.41
0.50	87.04	98.24	91.70	97.72	99.78	93.62
0.60	85.44	98.28	89.44	97.79	96.43	93.81
0.70	84.26	98.32	87.75	97.85	93.92	93.99
0.80	83.60	98.36	86.45	97.91	91.96	94.16
0.90	82.64	98.39	85.40	97.96	90.38	94.32
0.99	82.10	98.42	84.63	98.01	89.21	94.46

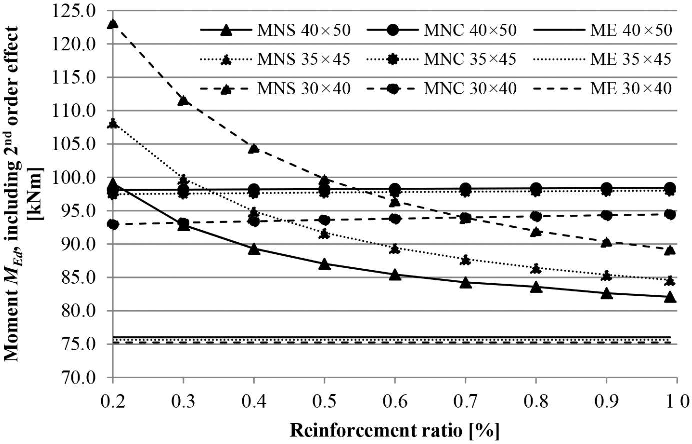


Fig. 5. Moments with the second order effects values determined with MNC and MNS in dependence of the reinforcement ratio and column cross – section. ME – equivalent moment

4.3. Influence of reinforcement ratio and relation of the force N_{Ed}/N_{Rd} on the second order effects in MNS and MNC

Data for the analysis of the column in the braced structure are similar to p. 4.2 and Fig. 2: the bending moment 150 kNm in the top node and 60 kNm in the bottom one. Material characteristics: concrete grade as C30/37 and steel RB500W. The cross-section of the beam is 30 × 60 cm and the distances between the faces of its supports are 5.95 m and 3.00 m. The cross-section of the column is 40 × 50 cm and its height is 4.10 m. The support flexibility of the node /1/ k_1= 0.1 and of the node /2/ was calculated from the relation (11). The effective creep coefficient was φ_{ef}= 1.32 and c_0 = 8. The reinforcement ratio ranges from 0.25% to 0.99%. Relation N_{Ed} to the design axial resistance of section N_{Rd} also differs. Results are presented in Tables 4 and 5 and on the Fig. 8.

Table 4

Moments with the second order effects calculated with MNS in relation to the reinforcement ratio and a longitudinal force value of a column in a braced structure

MOMENT M_{Ed} INCLUDING 2nd ORDER EFFECT [kNm] – MNS						
reinforcement ratio [%]	**relation between N_{Ed} and N_{Rd} force [%]**					
	1	**20**	**40**	**60**	**80**	**99**
0.25	66.9	82.8	98.6	113.8	128.5	142.8
0.50	66.6	78.4	90.9	103.6	116.3	129.0
0.75	66.5	77.0	88.3	99.8	111.6	123.5
0.99	66.5	76.4	87.2	98.2	109.6	121.1

Table 5

Moments with the second order effects calculated with MNC in relation to the reinforcement ratio and a longitudinal force value of a column in a braced structure

MOMENT INCLUDING 2ND ORDER EFFECT [kNm] – MNC						
reinforcement ratio [%]	**relation between N_{Ed} and N_{Rd} force [%]**					
	1	**20**	**40**	**60**	**80**	**99**
0.25	66.9	84.0	101.4	107.3	105.2	95.3
0.50	66.9	85.0	102.6	108.7	106.8	96.7
0.75	67.0	86.0	103.8	110.2	108.3	98.1
0.99	67.0	86.9	104.9	111.6	109.8	99.5

The key parameter in MNC is the K_r coefficient, which takes into account the decrease of the curvature for higher axial force values (Fig. 6). This coefficient should not exceed 1 (EC2 [17]). If $n < n_{bal}$, calculated K_r coefficient is higher than 1, the assumption that $K_r = 1$ should be made. The curvature value $1/r$ is constant. If $n_{bal} = n$, the coefficient $K_r = 1$, while if $n_{bal} < n < n_u$ the K_r coefficient is lower than 1 and the curvature $1/r$ decreases to zero. Values of the second order effect also decrease to zero, and the gradual closing of the cracks contributes to the reduction of their amount. This impact can also be observed in the results of analyses using MNC (Tab. 5 and Fig. 7).

For analysed columns, for the initial range of N_{Ed}/N_{Rd} (Fig. 7), values of the moments with the second order effects increase with an increase of the reinforcement ratio and an increase of the relation N_{Ed}/N_{Rd}, and differences between MNS and MNC are slight. With the increase of an effort, the results obtained from both methods start to diverge. Differences between MNS and MNC increase with an increase of the reinforcement ratio to relation $N_{Ed}/N_{Rd} \approx$ 50% for the reinforcement ratio ρ = 0.25% and $N_{Ed}/N_{Rd} \approx$ 80% for ρ = 0.99% (higher values of the moments with second order effects obtained initially from MNC). When the effort of

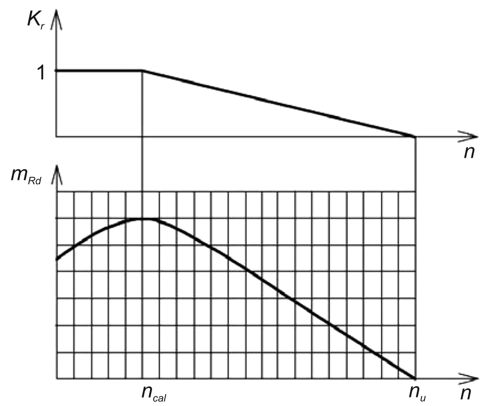


Fig. 6. K_r coefficient in relation to interaction curve $m_{Rd} - n$ according to [6]

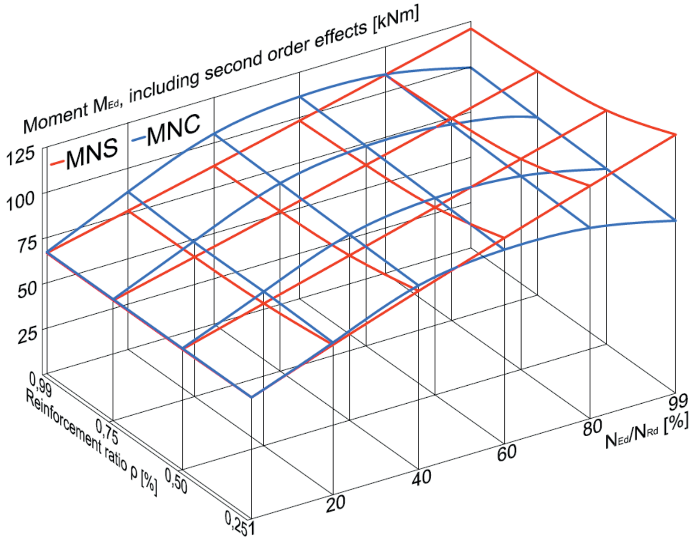


Fig. 7. Moments with the second order effects in dependence of the reinforcement ratio ρ and longitudinal force value, calculated with MNS and MNC

the column increases, the differences between values from both methods are greater (but they decrease with an increase of the reinforcement ratio). Then higher values of the moments with second order effects are obtained from MNS. It is worth mentioning that according to [12], the coefficient which causes deflection of compressed member to increase ($1/[1-(N_{Ed}/N_B)]$) as in equation (2) is exactly (up to 2 percent) for values of N_{Ed} below 60 per cent buckling force N_B.

In the analysed range of the reinforcement ratio in column I (in the braced structure), the biggest differences between moments with second order effects, calculated with MNS and MNC, are obtained when the effort of the column is the biggest and those differences equal from 20 to 50 percent.

5. Conclusions

Eurocode 2 and Polish standards statements do not indicate any criteria for the choice of a proper method for the calculation of second order effects. Results of moment M_{Ed}, which can be obtained from both methods, differ significantly from what has been presented in the conducted comparative analysis. The range of the presented analyses does not allow for the formulation of some general conclusions, however, the remarks are compatible with observations by the Authors of [5, 7, 14], quoted in p.1. MNC provides the second order effects closer to the experimental data [5, 7], however, attention should be paid to the statement 5.8.8.3 of Eurocode 2 [17], which restricts using MNC only for columns with a symmetrical, constant cross-section and symmetrical reinforcement.

The from presented results imply that, stiffness reduction factor considering cracking of beams in braced structures raise the second order effects, however not considerably. From the presented literature and the author's own analysis it can be concluded that the above mentioned factor can be assumed to be about 50%. In unbraced structures, the influence of this coefficient is significantly larger and a real, exact cracking of the beam should be considered. In both cases, when there is no necessity for high accuracy of calculations, cracking of a column, which decreases the second order effects, can be omitted.

The reinforcement ratio influences the second order effects in a braced structure only in MNS (the second order effect decreases with the increase of the reinforcement ratio), while in the MNC there is no relation. The highest second order effects calculated with MNS are obtained for columns with the smallest cross-section and they decrease with the increasing dimensions of the column's cross-section. In MNC, the situation is the opposite, the highest values of the second order effects are for columns with the largest cross-section, but the differences were not as significant as in the MNS.

The values of the second order effects depend on the longitudinal force. In MNC, they initially grow and then the influence of closing cracks can be observed as the second order effects decrease. In MNS, the second order effects increase consistently. In the presented analysis, the values of M_{Ed}, which were obtained from both methods, differ even by 50%.

The conducted analysis of a braced structure reveals some inconsequences, caused by Eurocode 2 statements. For columns with a slenderness even 3 or 4 times lower than the limit slenderness, the calculated second order effects were 13–17% and 23–28% for MNS and MNC, respectively. Therefore, as exceeding 10% they should not be omitted. For columns with slenderness closer to the limit values, the second order effects are distinctly higher.

The comparative analysis proves that the choice of method for evaluating the second order effects can have a considerable impact on the results of calculations of members subjected to compression. When both simplified methods (MNS and MNC) are allowed to be used in order to calculate the second order effects in Poland, even though they generate different results, some comments which limit the range of the usages should be formulated. This can help designers choose the right method.

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