

ZENON WASZCZYSZYN*, EWA PABISEK**, JOANNA KALISZUK***

HYBRID COMPUTATIONAL SYSTEMS
IN STRUCTURAL MECHANICSHYBRYDOWE SYSTEMY OBLICZENIOWE
W MECHANICE KONSTRUKCJI

Abstract

The first problem discussed in the paper is related to the reliability of structures. The simulation of the ultimate load of a steel girder is analyzed by means of a hybrid computational system FEM & ANN & p-EMP. The system consists of three components, with a low fusion grade. FEM is applied for 'off line' computing of the patterns for ANN training and testing. The trained ANN is then used for very fast generation of MC trials for the hybrid Monte Carlo method (HMC). The second problem corresponds to the identification of a neural material model (NMM) in elasto-plastic plane stress problems. The autoprogressive method (APM) was applied in a formulated hybrid system FEM/NMM/p-EMP with a very high fusion grade of components. The 'on line' interaction of all the components is applied at each load incremental step. In the third part of the paper the standing seminar on the application of ANNs in civil engineering, inspired by the ideas of the famous Professor Życzkowski's Seminar on applied mechanics, is briefly described.

Keywords: hybrid computational systems, finite element method (FEM), artificial neural network (ANN), neural material model (NMM), pseudo-empirical data (p-EMP), hybrid Monte Carlo method (HMC), autoprogressive method (APM), standing seminar

Streszczenie

Pierwszy problem, analizowany w tym artykule, dotyczy analizy niezawodności konstrukcji. Nośność graniczna dźwigara stalowego jest symulowana za pomocą hybrydowego systemu obliczeniowego FEM & p-EMP. FEM jest stosowana do obliczania wzorców uczących i testujących ANN. Nauczona sieć służy do szybkiego generowania pseudolosowych próbek w symulacjach hybrydowej metody Monte Carlo (HMC). Drugi problem odnosi się do identyfikacji neuronowego modelu materiału ekwiwalentnego (NMM) w wybranych problemach płaskiego stanu naprężeń. Zastosowano system hybrydowy FEM/NMM/p-EMP charakteryzujący się bardzo wysokim stopniem integracji użytych komponentów. Do identyfikacji NMM zastosowano metodę autoprogresywną (APM), która opiera się na interakcji 'on line' wszystkich komponentów na każdym przyroście obciążenia. Trzecia część pracy jest poświęcona stałemu seminarium nt. stosowania ANNs w inżynierii lądowej, inspirowanego przez słynne Seminarium Profesora Życzkowskiego z zakresu mechaniki stosowanej.

Słowa kluczowe: hybrydowy system obliczeniowy, metoda elementów skończonych (FEM), sztuczna sieć neuronowa (ANN), neuronowy model materiału (NMM), dane pseudopomiarowe (p-EMP), hybrydowa metoda Monte Carlo (HMC), metoda autoprogresywna (APM), stałe seminarium

* Prof. dr hab. inż. Zenon Waszczyszyn, Katedra Mechaniki Konstrukcji, Wydział Budownictwa i Inżynierii Środowiska, Politechnika Rzeszowska.

** Dr hab. inż. Ewa Pabisek, Instytut Technologii Informatycznych w Inżynierii Lądowej, Wydział Inżynierii Lądowej, Politechnika Krakowska.

*** Dr inż. Joanna Kaliszuk, Instytut Budownictwa, Wydział Inżynierii Środowiska, Uniwersytet Zielonogórski.

1. Introduction

Computer aided methods are a firm basis for the analysis of complex problems of structural mechanics. From among many new computational methods, the hybrid systems are worth emphasising. The main idea is to fuse components of different but compatible features, joining direct and inverse analysis, and the most important aim is to increase the efficiency of computer aided analysis. That is why the hybrid systems have been extensively developing in a research group under supervision of Z. Waszczyszyn, cf. [1].

Because of lack of time and scope of the paper, we have decided to focus on the analysis of only two problems which show potential possibilities of hybrid analysis. In what follows we discuss fusing of hard and soft methods components, i.e. FEM (Finite Element Method) and ANNs (Artificial Neural Networks). This selection seems to be reasonable since FEM is the best one in the direct (forward) analysis and ANNs are very efficient in simulation and identification analysis of data.

We present the extreme categories of integration grades, i.e. I) low fusion grade FEM&ANN&p-EMP, II) very high fusion grade FEM/ANN/EMP, where EMP corresponds to empirical or pseudo-empirical data. In I) only 'off-line' mode of computation is performed, vs. II), where 'on-line' computation mode is applied.

In order to illustrate extraordinary efficiency of the category I approach, the reliability analysis of a steel girder is presented. This example was taken from J. Kaliszuk's PhD. dissertation, defended 'cum lauda' [2]. The other example is related to the identification of NMM (Neural Material Model) for plane stress boundary value problems. It is taken from a chapter of E. Pabisek's postdoctoral dissertation [3], also defended 'cum lauda'.

Both examples have in fact a much wider context. The first example refers to the development of the Hybrid Monte Carlo (HMC) method, suggested by M. Papadrakakis et al. [4]. The other example concerns the creative modifications of the Auto-Progressive Method (APM), suggested in paper [5] by J. Ghaboussi.

At the end of the paper a short description of the activity of the Standing Seminar of ANNs Applications in Civil Engineering is briefly described, as an example of Professor Michał Życzkowski's inspiration.

2. HMC in reliability analysis of a steel girder

Monte Carlo methods are commonly applied in the reliability analysis of structures. The main problem of MC methods lies in simulation of trials. In case of engineering structures the trials are computed by means of FEM. The reliability is usually related to the structure ultimate load, which should take into account with a great amount of various parameters. Stationary type structural problems are defined with respect to the reliability of structures, measured by the probability of reliability

$$p_r \equiv Q = \text{Prob} \{G(R, S) > 0\} \equiv \text{Prob} \{R > S\} = \int_{G(\mathbf{X}) > 0} f(\mathbf{X}) d\mathbf{X} \quad (1)$$

where $G(\mathbf{X})$ – limit state function; R – resistance of structure; S – actions (loads) applied to the structure, $\mathbf{X} = \mathbf{X}^R + \mathbf{X}^S$ – random state variables.

The MC simulations correspond to computation of the integral in (1). Following the law of large numbers the Classical Monte Carlo estimator of the reliability probability is

$$\bar{p}_r = \frac{1}{NMC} \sum_{i=1}^{NMC} I(X) \quad \text{and} \quad I(\mathbf{X}) = \begin{cases} 1 & \text{for } G(\mathbf{X}) \geq 0, \\ 0 & \text{for } G(\mathbf{X}) < 0, \end{cases} \quad (2)$$

where: NMC – number of MC trials.

Now, let us evaluate the reliability of a girder shown in Fig. 1. It is made of steel with yield point $R_e = 235$ MPa and elasticity modulus $E = 205$ GPa. The girder is subjected to the action of uniform load $S = P$ and the resistance of the structure corresponds to the ultimate load $R = \lambda_{ult} P^*$, where $P^* = 200$ kN/m is the reference load, see [6].

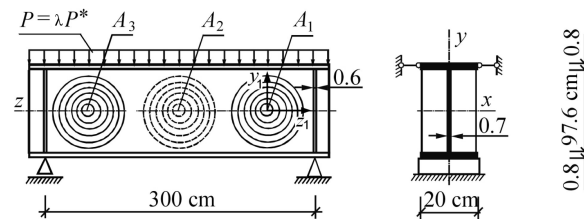


Fig. 1. Steel girder of I cross-section

Rys. 1. Dwuteowy dźwigar stalowy

Initial imperfections of the web plate are modeled as three smooth surfaces of the form

$$w_k(y_1, z_1) = A_k \cos(\pi y_1/B_y) \cos(\pi z_1/L_z), \quad (3)$$

where: A_k – amplitudes of imperfections; $B_y = L_z = 97.6$ cm – ranges of imperfections.

It was assumed that the imperfections can randomly appear in three equidistant areas $B_y \times L_z$. The amplitudes A_k are random variables of Normal probability density function (Npdf) with parameters: $\mu_{Ak} = 0$ mm and $\sigma_{Ak} = A_{ult}/2 = 3.5$ mm, where $A_{ult} = 7$ mm is the admissible value according to the Polish standard [7].

According to the hybrid approach, the training and testing patterns were computed by FEM. The nonlinear module of the COSMOS/M system [8] was applied. An elastic-plastic material with the yield surface was adopted assuming isotropic linear strain-hardening with HMH with $E_p = 0.0001 E$, where E_p is plastic stiffness. All parts of the girder (web, flanges and stiffeners) were covered by regular rectangular meshes of FEs of the type SHELL4T with 24 DOF. The total number of FEs was 1616. After preliminary computations the displacement control was used assuming 60 steps $\Delta v_0 = 0.01$ mm to compute the displacement $v_0 \in [0, 60]$ mm of the web centre, measured along the y axis.

The training patterns were computed for the input data placed regularly in the 3D-cube of coordinates $A_k \in [-3\sigma_{Ak}, 3\sigma_{Ak}]$. Assuming 5 points on the A_k axes, the number of training patterns equals $L = 3^5 = 125$. The set of $T = 100$ testing patterns was randomly selected as 100 points in the 3D-cube of variables A_k , assuming Npdf with the same parameters as for the training patterns.

In Fig. 2 the equilibrium path $\lambda(v_0) \in [1.180, 1.393]$, computed for input data $A_1 = -0.525$ cm, $A_2 = 1.05$ cm, $A_3 = 1.05$ cm, is shown. The ultimate state of the girder corresponds to the load parameter $\lambda^G = \lambda_{\min}^G = 1.180$. This state is related to the overall instability of the girder caused by buckling of the upper flange and web plate. In case of a perfect girder, i.e. for $A_1 = A_2 = A_3 = 0$, the ultimate load parameter is $\lambda_{\text{perf}}^G = 1.248$, and for the initial imperfections $A_1 = -1.05$ cm, $A_2 = 0.525$ cm, $A_3 = -1.05$ cm the ultimate load corresponds to $\lambda^G = \lambda_{\max}^G = 1.393$. The average CPU time to compute one pattern was about 300 sec.

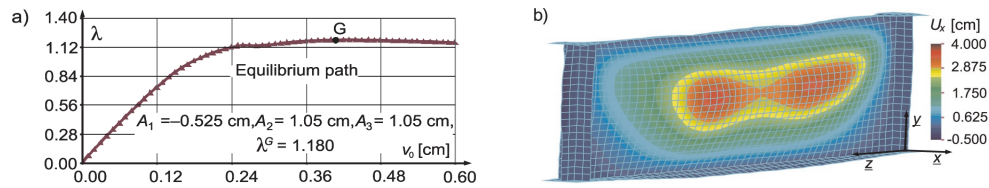


Fig. 2: a) Equilibrium path, b) displacements of girder at load factor $\lambda^G = 1.180$

Rys. 2: a) Ścieżka równowagi, b) przemieszczenie dźwigara dla parametru obciążenia $\lambda^G = 1.180$

At the second stage of the hybrid approach FEM&ANN the standard, feed-forward, two layered ANN was formulated. The following input vector and scalar output were applied

$$\mathbf{X} = \{A_1, A_2, A_3\}, \quad y = \lambda_{\text{ult}} \quad (4)$$

to formulate the sets of training and testing patterns

$$\mathcal{L} = \{(\mathbf{X}, t_p) \mid p = 1, \dots, L\}, \quad \mathcal{T} = \{(\mathbf{X}, t_p) \mid p = 1, \dots, T\}, \quad (5)$$

where: t_p – target output computed by FEM; $L = 125$, $T = 100$ – numbers of training and testing patterns, respectively.

The MATLAB Neural Network Toolbox [9] was explored. The Standard Neural Network (SNN) of structure SNN: 3–H–1 with sigmoidal hidden neurons and linear output was designed using the cross-validation procedure, see [10]. The Levenberg-Marquardt learning method was used and after extensive cross-validation the number of hidden neurons $H_{\text{opt}} = 8$ was evaluated.

The accuracy of the designed network was evaluated by relative errors

$$\text{avr } epV = \frac{1}{V} \sum_{p=1}^V ep, \quad \max epV = \max_p ep \quad (6)$$

where $ep = (1 - y_p / t_p) \cdot 100\%$ – relative error for the p -th pattern; $V = L$, T – the numbers of training and testing patterns. Another estimation is given by statistical parameters, i.e. by standard error $St\varepsilon V$ and correlation parameter rV , cf. [10]. In case of the trained network SNN: 3-8-1 the errors are: $\text{avr } epL \approx \text{avr } epT = 0.77\%$, $\max epL \approx \max epT = 3.90\%$, $St\varepsilon L \approx St\varepsilon T = 0.0136$, $rL = 0.959$, $rT = 0.790$.

The designed network SNN: 3-8-1 was used for the simulation of MC trials. First of all, it was checked that for computing of 10^8 MC trials (such a great number of MC trial corresponds to the 3σ bar normal distribution with the approximation error less than 1%). The network

consumed 416 sec. of CPU time for the simulation of 10^8 trials. This time is comparable with 300 sec. needed for the computation of one pattern by the FEM system COSMOS/M. Next, the network was used for the computation of discrete points at the reliability curves $\bar{Q}(\bar{P})$, where \bar{Q} is the probability of reliability (1) and $\bar{P} = S$ is the load applied to the girder. In the definition of the reliability curve there are two cases corresponding to the assumption of the action variables: 1) Case 1: load \bar{P} is a random value and Npdf has parameters $\bar{P}_j = \mu_{P_j}$, $\bar{\sigma}_{P_j} = 0.1\bar{P}_j$. 2) Case 2: P_j is a deterministic real value. In Fig. 3 two curves corresponding to both cases are shown. It is worth mentioning that in Case 1 the reliability curve $\bar{Q}(\bar{P}_j)$ is smooth, without discontinuity type parts that occur in Case 2 of the curve $\bar{Q}(P_j)$

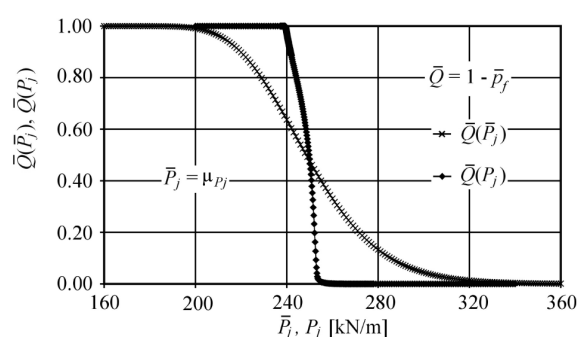


Fig. 3. Reliability curves for random loads \bar{P}_j and fixed (deterministic) loads P_j

Rys. 3. Krzywa niezawodności dla obciążeń losowych \bar{P}_j i deterministycznych P_j

In Table 1 there are listed CPU times corresponding to computation of the reliability curve $\bar{Q}(\bar{P}_i)$ for two numerical versions of CMC: 1) hybrid version FEM&SNN, 2) FEM is hypothetically used for the computer simulation of the same number of MC trials as in the hybrid version. The computations were performed by a PC with processor AMD ATHLON XP 2.4 1.3 GHz.

Table 1

CPU time for two numerical versions of CMC (Classical Monte Carlo method)

| Simulation of CMC trials by | | | |
|--|--|--|---|
| SNN: 3-8-1 | | FEM system COSMOS/M | |
| Operations | CPU sec. | Operations | CPU sec. |
| Preparation of 225 patterns by FEM $225 \times 300 = 19 \times 3600$ sec. | 67500 | Computation of one pattern | 300 |
| Training and testing of BPNN, ≈ 20 hrs = 20×3600 sec. | 72000 | — | — |
| Simulation of 10^8 CMC trials | 416 | Hypothetical computations of 10^8 trials | 300×10^8 |
| Total CPU time | 1.4×10^5 sec. | Total CPU time | 3.0×10^{10} sec. |

Application of the hybrid FEM&SNN method needs 1.4×10^5 sec. ≈ 39 hrs = 1.62 days. The hypothetical time of computing 10^8 CMC trials by the FEM system COSMOS/M equals about 3.0×10^{10} sec. $\approx 3.47 \times 10^5$ days. If we assumed hypothetically that we have at our disposal a very efficient numerical version of the MC method, in which we need only 10^4 trials computed by COSMOS/M, then the total time would be 300×10^4 sec. This gives the computation about 20 times longer than for the CPU time needed for the hybrid method.

3. APM as a hybrid FEM/NNM/EMP system

The AutoProgressive Method (APM) was formulated by J. Ghaboussi and his research group at Univ. Illinois, USA, see [5, 11, 12]. The main idea of APM lies in the interaction of all the components of the hybrid system FEM/NNM/EMP at each step of a modified Newton-Raphson method. Such an 'on-line' integration is based on a two-stage incremental N-R algorithm sketched in Fig. 4.

Stage I corresponds to performing the load increment n for which strains and stresses are computed at each Gauss integration point g in a FEs e , see Fig. 4a. Stage II is carried out for ${}^c_n\{\mathbf{I}_e^g, \mathbf{I}_\sigma^g\}^e$ correction of displacements at the each control point j . Respective strains and stresses, ${}^c_n\{\mathbf{II}_e^g, \mathbf{II}_\sigma^g\}^e$ computed in Stage II are shown in Fig. 4b. Using these sets of data new patterns are generated as sets ${}^c_n\{\mathbf{II}_e^g, \mathbf{I}_\sigma^g\}^e$, see Fig. 4c. At step n the newly generated patterns are included into the training set and the network NMM is retrained.

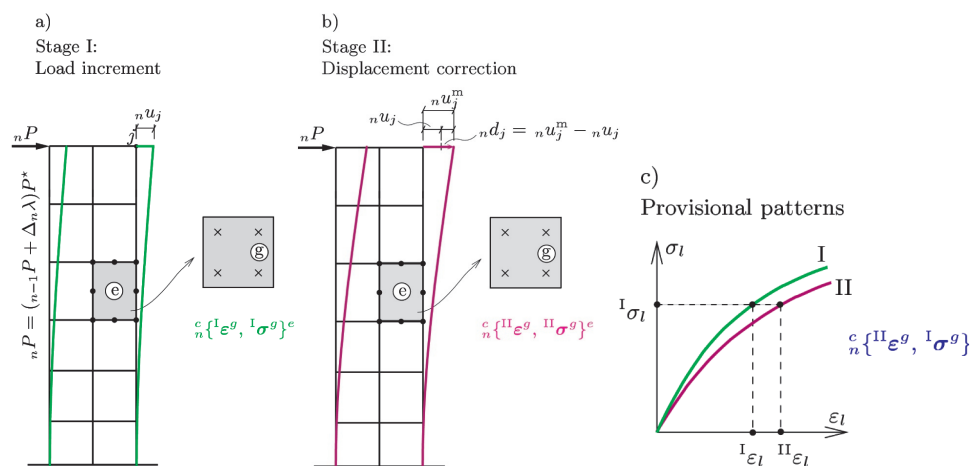


Fig. 4. Scheme of the modified Newton-Raphson method in APM: a) stage I – load increment, b) stage II – displacements correction, c) formulation of updated patterns

Rys. 4. Schemat zmodyfikowanej metody Newtona-Raphsona w APM: a) etap I – przyrost obciążenia, b) etap II – korekcja przemieszczeń, c) formułowanie uaktualnionych wzorców

A crucial point of APM is the identification of the Neural Material Model (NMM) of an equivalent material in the sense of the homogenization theory. The consistent incremental matrix \mathbf{K}_t of a hypo orthotropic material can be computed in an explicit form applying partial derivatives, see [3, 11]

$${}_{n+1}^c k_{lm} = \frac{\partial {}_{n+1}\Delta\sigma_r}{\partial {}_{n+1}\Delta\epsilon_s} \quad (7)$$

where: n – number of load increment, c – number of global iteration cycle, ϵ_s , σ_r for $s, r = 1, 2, 3$ – inputs and outputs of the network NMM. It was proved that the neural networks with two hidden layers should be applied to obtain satisfactory accuracy of the computed values of partial derivatives in (7). The values of derivatives depend on the given values of inputs and outputs and on the NMM parameters.

This means that the trained NMM is not ‘a black box’, but in fact it consists of a relationship of stress to strain and can be used for computing the material constitutive matrix.

The identification takes into account fulfilling of compatibility equations, cf. Fig. 4b

$${}_n d_j = ({}_n u_j^m - {}_n u_j) \leq \text{admed} \quad (8)$$

where: j – number of control points, ${}_n u_j^m$, ${}_n u_j$ – displacements measured and computed for the n load increment. The admissible small values of the displacement differences *admed* in (8) are reached after the whole loading process is repeated during a finite number of iteration cycles. In the continuation procedure, the incremental FE model is updated due to application of NMM as a numerical material procedure.

Selection of patterns for the training of NMM and cycle of loading $c = 1$ are crucial numerical problems of APM, cf. [3, 5]. The NMM identification is carried out for a selected load program. It was proved that the material network NMM can be generalized also for other load programs after retraining on NMM by means of other measurement data, see [3, 12].

Two boundary value problems, selected from [3, 13], are discussed below as examples of APM applications.

Example 1: Tension perforated strip

The strip shown in Fig. 5 was investigated by Zienkiewicz et al. in 1969 in paper [14], and then it has been used by many authors as a bench-mark test for verification of their results. In paper [15] by Waszczyszyn and Pabisek, the hybrid program of low degree of fusion FEM/ANN was also discussed.

Because of double symmetry only a quarter of the strip was analyzed. This part of strip was covered by a mesh composed of 72 eight-node isoparametric FEs. The elastic-plastic material with the HMM yield surface and isotropic linear strain hardening was assumed adopting mechanical parameters shown in Fig. 5b. The equilibrium path $\lambda(u_A)$ was computed by means of the ANKA FE code [16]. This loading path was assumed to be pseudo-empirical curves, shown in Fig. 6.

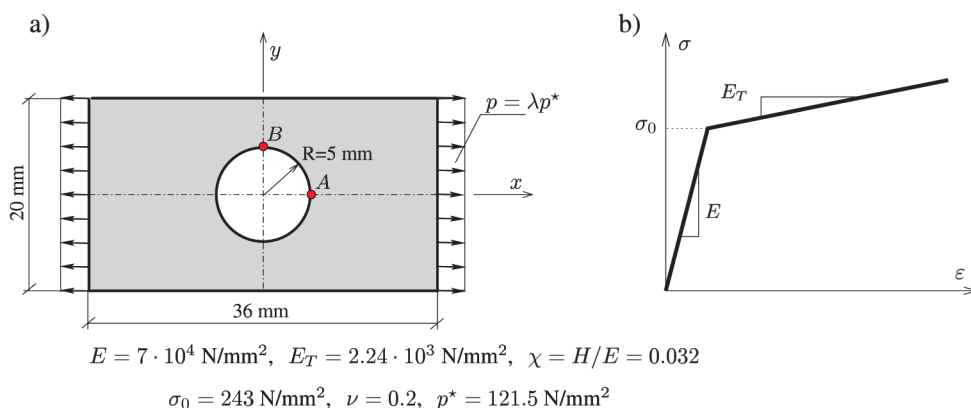


Fig. 5a) Geometry and load data, b) mechanical characteristics for elastoplastic material with isotropic linear strain-hardening

Rys. 5a) geometria i obciążenie, b) mechaniczne charakterystyki dla materiału sprężysto-plastycznego z liniowym, odkształceniowym wzmocnieniem

The network SSN:3-15-15-3 was formulated. Application of the APM led to only two cycles for identification of the material model NMM, see Figs 6. The identification proved to be quite accurate, as shown in Fig. 6b, where another equilibrium path $\lambda(v_B)$ was applied for the verification of the identified material model.

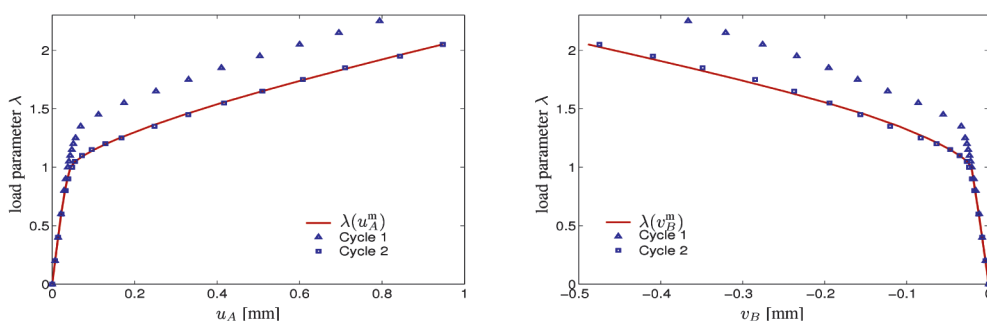


Fig. 6. Pseudo-empirical loading curves $\lambda(u_A)$ and equilibrium points evolution during the training of NMM: 3-15-15-3

Rys. 6. Pseudo empiryczne krzywe $\lambda(u_A)$ i punkty równowagi podczas uczenia sieci NMM: 3-15-15-3

In Fig. 7a, the distribution of effective stresses $\sigma_e(x,y)$ is shown for the load parameter $\lambda = 2.0$. These stresses were computed by means of an FEM procedure assuming a perfect elasto-plastic material of parameters shown in Fig. 6b. The effective stress distribution was then computed by means of identified material model NMM, cf. Fig. 8b. Quite good approximation of stress distributions is shown in Figs. 7a and 7b.

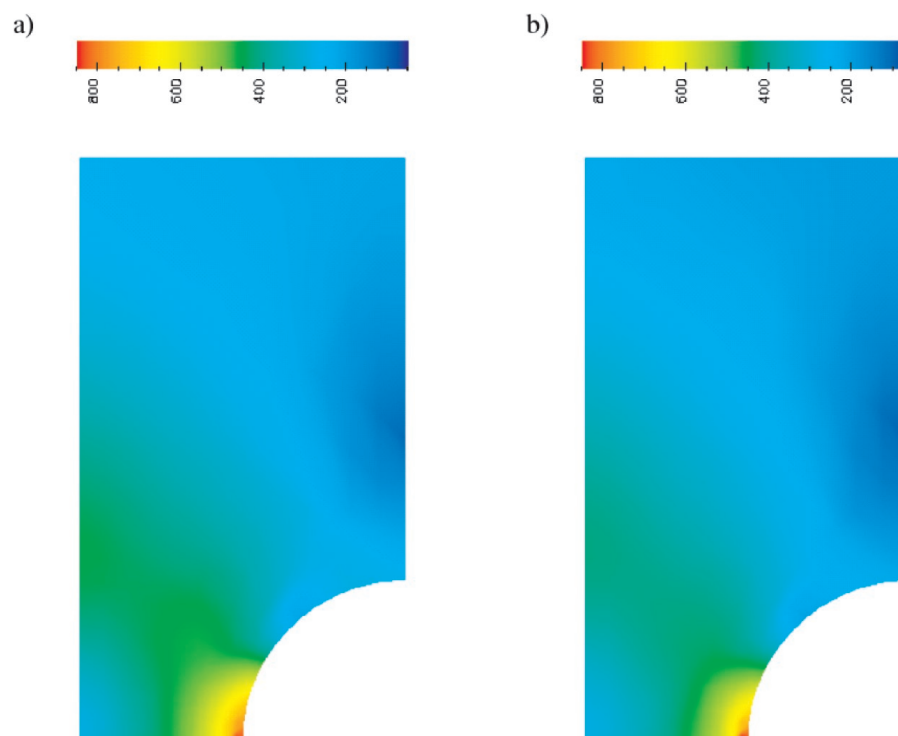


Fig. 7. Distribution of reduced stresses $\sigma_e(x, y)$ for load parameter $\lambda = 2.0$, computed for: a) FEM using elastic-plastic material and b) material identified by NMM and hybrid system FEM/NNM/EMP

Rys. 7. Rozkład naprężeń zredukowanych $\sigma_e(x, y)$ dla parametru obciążenia $\lambda = 2.0$, obliczone dla: a) FEM przyjmując materiał sprężysto-plastyczny, b) materiał identyfikowany przez NMM i system hybrydowy FEM/NNM/EMP

Example 2. Notched beam

This boundary value problem, shown in Fig. 8a, was called in [14] “a notched beam”. It is a symmetric, simple supported strip, with a notch around point B and loaded by concentrated forces applied to points A.

The “beam” was analyzed by the same FE program and types of FEs as in Example 1. A half of the beam area was covered by 89 eight-node isoparametric FEs. Because of much more complicated stress distribution, the neural model NMM: 3-15-15-3, trained on the tension strip model, was retrained. Besides the plane tension stress distribution the network was also subjected to compression. The obtained NMM_{ret} was verified on Example 1 and then the retrain neural model was applied to the FEM/ NMM_{ret} . The graphics shown in Fig. 8b point out that the hybrid approximation gives quite good fitting of the predicted points to the pseudo-empirical loading curves $\lambda(v_A)$ and $\lambda(v_B)$.

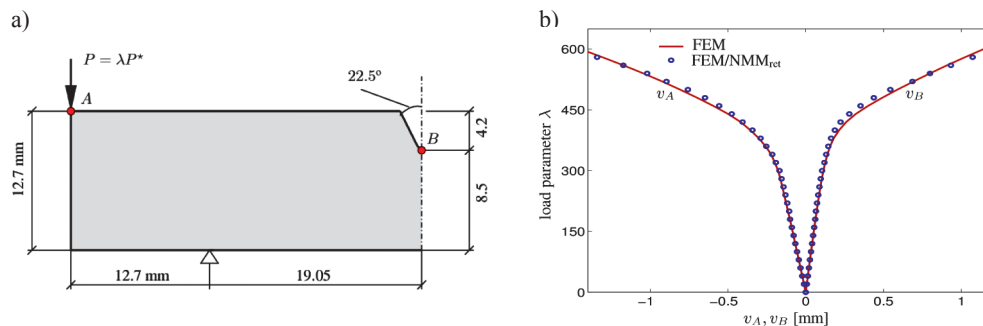


Fig. 8: a) Strip made of the same material as the tension strip, b) pseudo-empirical curves $\lambda(v_A)$ and $\lambda(v_B)$ computed by FEM program and loading points computed by hybrid FEM/NMM_{ret}

Rys. 8: a) Tarcza wykonana z tego nowego materiału jak tarcza na rys. 5, b) pseudoempiryczne krzywe $\lambda(v_A)$ i $\lambda(v_B)$ obliczone przez program FEM i punkty obciążenia obliczone przez system hybrydowy FEM/NMM_{ret}

4. Instead of the end: Standing Seminar on ANNs Applications in Civil Engineering

Two first authors of this paper were closely collaborating with Professor Michał Życzkowski. Prof. Z. Waszczyszyn was his PhD student No.4, and in his remembrances [17] he mentioned how strongly he was fascinated by the famous Życzkowski Seminar. Many years after that, about 1995, prof. Z. Waszczyszyn decided to focus his research interest on the computational methods of artificial intelligence and, especially, on ANNs. Collecting his research group at the Institute of Computer Methods in Civil Engineering, prof. Z. Waszczyszyn decided to arrange a permanent seminar, having in mind the famous Życzkowski Seminar. The Standing Seminar on Applications of ANNs in Civil Engineering (called in short Seminar) started in October of the winter semester 1997/98 and has been continued until now. In this Seminar prof. Z. Waszczyszyn's activity was strongly supported by a Professor Życzkowski's student, now Assoc. prof. E. Pabisek. We were firmly encouraged by Professor Życzkowski's advice how to start with the Seminar.

The main idea of the Seminar was to develop a new research tool, like ANNs, and apply it in many fields of civil engineering. A very important point was to have a discussion 'forum', similar to that arranged by our Master Professor Życzkowski. The Seminar has been rapidly developed. After a short time about 30 participants started with ANNs learning and tried to use them in the analysis of various civil and structural engineering problems.

The participants have been collected from 7 Polish technical universities from Cracow, Rzeszów, Zielona Góra, Wrocław, Łódź and Białystok. The topics were taken from the PhD theses and postdoctoral dissertations, as well as from various applications of ANNs in the analysis of problems at the participants' universities. In such a way several small interinstitute and inter-university teams have started, based on financial support of projects and grants submitted from various research places. Ten years of activity were discussed briefly in paper [1].

The most important event seems to be related to the Professor Subsidy of the Polish Science Foundation awarded to Z. Waszczyszyn in 2001. In the frame of this Subsidy, he could offer

five scholarships to young participants of the Seminar. Another fruitful activity is connected with arrangement of special courses and schools devoted to the development of ANNs applications. From among these activities, coordination of two CISM (Centre International des Sciences Mécaniques) Advanced Schools by Z. Waszczyszyn, delivering lectures and participation of the Seminar members in three CISM Schools are worth mentioning. Till now 4 postdoctoral dissertations and 14 PhD theses have been written, based on 14 scientific projects and grants. The publishing activity covers 7 scientific monographs, 23 chapters in books, keynote lectures and state-of-the-art papers. Publishing over 40 papers and delivering about 80 presentations at the international conferences also deserve a mention.

We think that we have made some small efforts to be closer to Professor Życzkowski's ideas of service to science, to the truth and to people. Thus, we hope that we can dedicate the discussed Seminar to Professor Życzkowski since it has tried to follow his inspirations and ideas.

References

- [1] Waszczyszyn Z., *Artificial neural networks in civil and structural engineering: Ten years of research in Poland*, Computer Assisted Mech. Eng. Sci., 13, 2006, 489-512.
- [2] Kaliszuk J., *Reliability Analysis of Structures by Artificial Neural Networks* (in Polish), Ph.D Th., University of Zielona Góra 2005.
- [3] Pabisek E., *Hybrid Systems Integrating FEM and ANNs for the Analysis of Selected Problems of Structural and Materials Mechanics* (in Polish), Monographs 369, Series Civil Engineering, Postdoctoral dissertation, Faculty of Civil Eng., Cracow University of Technology 2008.
- [4] Papadrakakis M., Papadopoulos V., Lagaros N.D., *Structural reliability analysis of elastic-plastic structures using neural network and Monte Carlo simulation*, Comp. Meth. Appl. Mech. Eng., 136, 1996, 145-163.
- [5] Ghaboussi J., Pecknold D.A., Zhang M., Haj-Ali R.M., *Auto-progressive training of neural constitutive models*, Int. J. Num. Meth. Eng., 42, 1998, 105-126.
- [6] Kaliszuk J., Waszczyszyn Z., *Reliability analysis of a steel girder by the hybrid FEM/BPNN Monte Carlo Method*, in: M.A. Gizejewski et al. (Eds), *Progress in Steel, Composite and Aluminum Structures*, Balkema, Taylor&Francis Group, London 2006, 346-347.
- [7] PN-B-06200.2002, *Building steel structures. Requirements for production and control. Basic requirements*.
- [8] COSMOS/M, *Finite element analysis system. Version 2.5*. Los Angeles, California: Structural Research and Analysis Corp. 1999.
- [9] *Neural Network Toolbox for use with MATLAB. User's Guide, Version 3*, The Math Works inc, Natick, MA, USA 1998.
- [10] Waszczyszyn Z., *Fundamentals of artificial neural networks*, Ch. 1 in: Z. Waszczyszyn (Ed.), *Neural Networks in the Analysis and Design of Structures*, CISM Courses and Lectures, No. 404, Springer, Wien-New York, 1998, 1-51.
- [11] Hashash Y.M., Jung S., Ghaboussi J., *Numerical implementation of a neural network based material model in finite element analysis*, Int. J. Num. Meth. Eng., 59, 2004, 989-1005.

- [12] Ghaboussi J., *Advances in neural networks in computational mechanics and engineering*, Ch. 4 in: Z. Waszczyszyn (Ed.), *Advances of Soft Computing in Engineering*. CISM Courses and Lectures, Vol. 512, Springer, Wien New-York 2010, 191-236.
- [13] Pabisek E., *Self-learning FEM/ANN approach to identification of an equivalent material models on measured structural constraints*, *Comput. Assis. Mech. Eng. Sci.*, 15, 2008, 67-78.
- [14] Zienkiewicz O.C., Valliappan S., King I.P., *Elasto-plastic solutions of engineering problems: 'Initial stress', finite element approach*, *Int. J. Num. Meth. Eng.*, 1, 1969, 75-100.
- [15] Waszczyszyn Z., Pabisek E., *Hybrid NN/FEM analysis of the elastoplastic plane stress problem*, *Computer Assisted Mech. Eng. Sci.*, 6, 1999, 177-188.
- [16] Waszczyszyn Z., Cichoń Cz., Radwańska M., *Stability of Structures by Finite Element Methods*, Elsevier, Amsterdam 1994.
- [17] Waszczyszyn Z., *Master – my encounters with Professor Życzkowski* (in Polish), Nasza Politechnika, 2007/3, Cracow Univ. of Technology, 2007.