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SELECTED ASPECTS INVOLVED IN THE OPTIMIZATION
OF CRANES WITH PIVOTING BOOMS

WYBRANE ZAGADNIENIA OPTYMALIZACJI ŻURAWI
Z WYCHYLNYM WYSIĘGNIKIEM

Abstract

Certain crane operations – hoisting/lowering the payload connected to a slewing jib – require a nonlinear description to take Euler and Coriolis forces into account, the impacts of which should already be minimised at the stage of selection of the system parameters and mechanism structure. Optimal sets of parameters for the crane mechanisms thus obtained were optimised for the full range of slewing motion. It is demonstrated that the selection of the geometric dimensions of the structural elements of the hoisting mechanisms, i.e. the slewing system and counterbalances, enables the horizontal track error load to be minimised whilst the forces acting on the mechanism and inducing its vertical movement can be reduced. Thus, for the assumed lifting capacity and distance jaunt we get the structure of the crane mechanism that guarantees the minimal consumption of energy.

Keywords: cranes, counterbalance, pivoting jib, slewing system, Euler's forces, Coriolis forces, energy consumption, nonlinear and non-stationary differential equations of motion, vibration, parametric optimisation, horizontal track error load, one-link jib crane

Streszczenie

Niektóre operacje żurawi: podnoszenie/opuszczanie ładunku połączone ze zwodzeniem wysięgnika wymagają nieliniowego opisu, biorąc pod uwagę siły Eulera i Coriolisa, których wpływ powinien być minimalizowany już na etapie doboru struktur i parametrów mechanizmów. Uzyskane w ten sposób optymalne zestawy parametrów mechanizmów żurawia zostały zoptymalizowane dla pełnego zakresu ruchu zwodzenia. Jak wykazano w pracy, poprzez właściwy dobór wymiarów geometrycznych elementów konstrukcji żurawia tworzących mechanizmy linowe: wypadu i przeciwwagi możliwe jest uzyskanie minimum błędu prostowodności poziomej toru ładunku oraz minimalizacja sił w mechanizmie zwodzenia. W efekcie dla założonych: udźwignięć i odległości wypadu otrzymujemy struktury mechanizmów żurawia gwarantujące najmniejsze zapotrzebowanie energii.

Słowa kluczowe: żurawie, przeciwwaga, wychylny wysięgnik, układ zwodzenia, siły Eulera, siły Coriolisa, zużycie energii, nieliniowe i niestacjonarne równania ruchu, drgania, optymalizacja parametryczna, błąd prostowodności poziomej, żuraw z wychylnym wysięgnikiem

1. Introduction

This study investigates the energy efficiency of jib lift mechanism structures with unilateral constraints (rope/cable mechanisms) optimized together with the jib-balancing mechanism.

Previous works on this subject [2, 3] were limited in scope as the analysis was mostly restricted to a few selected boom positions. More recent works such as [6, 7] focused on the search for the optimal position of blocks in a compensation mechanism such that the boom's unbalanced moment can be minimized. In work [6], a minimum deviation of the vertical load is sought for a finite number of boom positions, based on the linearised form of the objective function.

A similar problem (trajectory optimization) is investigated in [7] with respect to the trajectory optimization of a double-rocker four-bar mechanism.

Cranes with a pivoting jib are complex dynamic systems governed by nonlinear, non-stationary differential equations of motion [4, 5].

The study investigates the behaviour of a crane with a pivoting jib (the physical model of which is shown in Fig. 1) subjected to the applied loads: Q – lifting load due to hoisted mass, G_p – counterweight and G_w – weight of the jib. Respective forces acting on the ropes due to the lifting load – S_Q jib lifting – S_{w^w} counterweight – S_p act at acute angles to the jib: α, β, γ – not indicated in Fig. 1.

The physical model of a one-link crane is governed by the following equations of motion:

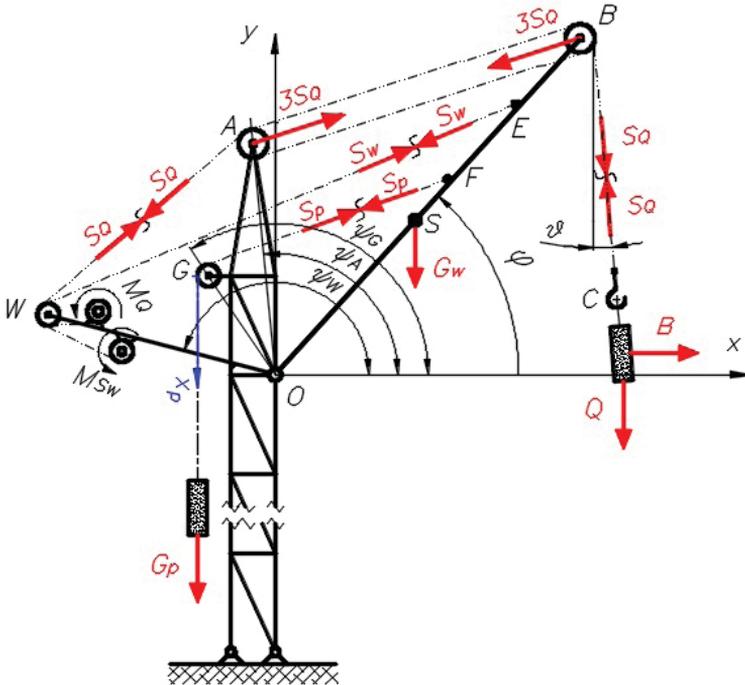


Fig. 1. Physical model of a one-link crane

$$\left\{ \begin{array}{l} J_{WO} \varepsilon_{\phi} = L_{OB} \left[3 \cdot \sin(\alpha) \cdot S_Q - \cos(\phi - \nu) \cdot S_Q + \kappa_{OE} \cdot \sin(\beta) \cdot S_W + \kappa_{OF} \cdot \sin(\gamma) \cdot S_P - \kappa_{OS} \cdot \cos(\phi) \cdot G_W \right] \\ m_Q \left[a_{BC} - L_{BC} \cdot \omega_{\nu}^2 - L_{OB} \sin(\phi - \nu) \cdot \varepsilon_{\phi} + L_{OB} \cos(\phi - \nu) \cdot \omega_{\phi}^2 \right] = \cos(\nu) \cdot Q - S_Q \\ m_Q \left[L_{BC} \cdot \varepsilon_{\nu} + 2 \cdot \omega_{\nu} \cdot v_{BC} - L_{OB} \cos(\phi - \nu) \cdot \varepsilon_{\phi} - L_{OB} \sin(\phi - \nu) \cdot \omega_{\phi}^2 \right] = -\sin(\nu) \cdot Q \\ m_P a_{XP} = G_P - S_P \end{array} \right. \quad (1)$$

where: $\varepsilon_{\phi}, \omega_{\phi}$ – angular acceleration and angular velocity of the jib, $\varepsilon_{\nu}, \omega_{\nu}$ – angular acceleration and angular velocity of the load Q , a_{BC}, v_{BC} – acceleration and velocity of the longitudinal motion of the load Q , a_{XP} – vertical acceleration of the counterweight, L_{OB} – length of the jib, $\kappa_{OE}, \kappa_{OF}, \kappa_{OS}$ – normalised with respect to L_{OB} distances: $\overline{OE}, \overline{OF}, \overline{OS}$.

The optimisation of mechanical structures such as that, which is required in order to minimise the operating dynamic forces and maximum energy uptake is categorised as a vibration isolation method, involving the reduction of the energy of the vibration source

For each problem involving the dynamic behaviour of investigated crane mechanisms, the specific optimisation task is formulated by defining the objective function:

- 1) slewing mechanism \rightarrow minimising the change of the payload level position – Δy , whilst the winch is immobile,
- 2) counter weight \rightarrow minimising the jib lifting work,
- 3) jib lifting mechanism \rightarrow minimum force in the rope winch jib.

2. Parametric optimisation of a slewing mechanism

The first step in the optimisation of mechanisms in a crane with a pivoting jib should involve the slewing mechanism. The optimisation procedure will determine the position of the peak pulley whilst the objective function is the minimisation (reduction) of the horizontal track error. The change of the slewing range is implemented by the slewing mechanism. It is vital that the slewing motion does not bring about a change to the payload level position.

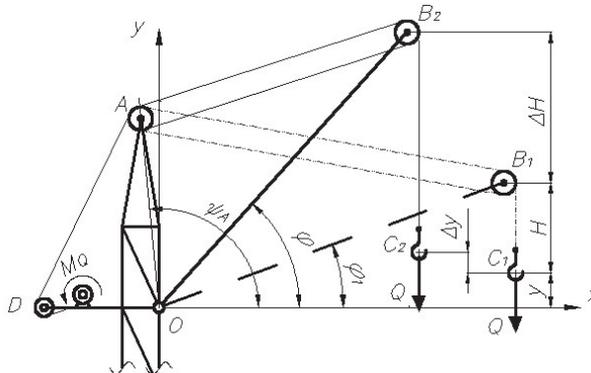


Fig. 2. Change of the jib's angular position in a one-link crane with a blocked winch

It is assumed that the hoisting load winch mechanism is blocked; thus, the rope length L from which the payload is suspended will not change during the hoisting phase.

In the case of extensible jibs, this condition can never be fully satisfied. The design of the slewing mechanism is considered satisfactory if the horizontal track error load during slewing – δ is less than 2%. The loads horizontal track error is understood as the absolute value of the ratio of the payload deviation from the straight-line trajectory to the total change of horizontal deviation:

$$\delta = \left| \frac{y(\phi_{\min}) - y(\phi_{\max})}{x(\phi_{\min}) - x(\phi_{\max})} \right| \cdot 100\% , \quad (2)$$

where: the angles ϕ_{\max} and ϕ_{\min} correspond to the lowest and the highest position of the jib, respectively.

Two configurations of the slewing mechanism represented by the jib's inclination angles ϕ_1, ϕ_2 are shown in Fig. 2. Thus, we write:

$$\begin{cases} i_w \cdot \overline{AB_1} + \overline{B_1C_1} = L \\ i_w \cdot \overline{AB_2} + \overline{B_2C_2} = L \\ L = const \end{cases} \quad (3)$$

The change of the jib's angular position involves a change of distance between the axes of the rope pulley B and the top pulley A , thus changing the length of the rope's free end section ΔL_{BC} on which the payload is suspended. On the other hand, when the pulley dimensions are omitted as negligibly small in relation to the distance between them, ΔL_{BC} can be derived from the formula:

$$\Delta L_{BC} = \Delta H - \Delta y = i_w (L_{AB1} - L_{AB2}), \quad (4)$$

where: i_w – transmission ratio of the compensating pulley block, L_{ABi} – distance between the rope pulleys A and B for an arbitrary i -th position of the jib (ie $L_{ABi} = \overline{ABi}$).

Comparing two arbitrary angular positions of the jib, we get the formula expressing the payload height:

$$\begin{cases} y_1 = L_{OB} \cdot \sin(\phi_1) - H \\ y_2 = L_{OB} \cdot \sin(\phi_2) - H - (\Delta H - \Delta y) \end{cases} . \quad (5)$$

The change of the payload position level Δy caused by varying the jib's angular position $\phi_1 \rightarrow \phi_2$ is given as:

$$\Delta y(\phi) = L_{OB} \cdot \sin(\phi) - L_{OB} \cdot \sin(\phi_1) - i_w [L_{AB}(\phi_1) - L_{AB}(\phi)], \quad (6)$$

$$\text{and: } L_{AB}(\phi) = \sqrt{L_{OA}^2 + L_{OB}^2 - 2L_{OA}L_{OB} \cdot \cos(\psi_A - \phi)}, \quad (7)$$

where: L_{OA} – distance between the rotation axis of the jib O and the top pulley A , ψ_A – angle of horizontal inclination of the line connecting the rotation axes of the jib O and the top pulley A (typically $\pi/3 \leq \psi_A \leq 2\pi/3$).

For a stabilised angular position of the jib ϕ_1 , we get:

$$\Delta y(\phi) = L_{OB} \left[\sin(\phi) - \sin(\phi_1) \right] - i_c \left[\sqrt{\kappa_{OA} + 1 - 2\kappa_{OA} \cos(\psi_A - \phi)} - \sqrt{\kappa_{OA} + 1 - 2\kappa_{OA} \cos(\psi_A - \phi_1)} \right] \quad (8)$$

where: $\kappa_{OA} = \frac{L_{OA}}{L_{OB}}$. (9)

In Fig. 1, the hook is attached directly to the end of the rope. In most cranes, an additional pulley block is connected between the jib top incorporating the pulley B and the hook, thus forming a sheave block with the transmission ratio $i_z > 1$. In this situation, the transmission ratio of the entire slewing mechanism becomes $i_c = i_w/i_z$, where $i_{c \min} = 3 [2, 3]$.

Thus, the formulated optimisation problem uses an objective function Δy , the decision variables being κ_{OA} and the angle ψ_A . The length L_{OB} and the transmission i_w (i_c) are taken as constant in the optimisation procedure. It is assumed that $L_{OB} = 30$ m, and the angle variability is in the range $\phi \in [15^\circ - 75^\circ]$.

Optimisation problem 1

The optimisation problem involves finding the optimal values of κ_{OA} and the angle ψ_A for which the quadratic functional $J(\kappa_{OA}, \psi_A)$ reaches its minimum, assuming that $\phi_1 = \phi_{\min}$.

$$J(\kappa_{OA}, \psi_A) = \int_{\phi_{\min}}^{\phi_{\max}} \left[\Delta y(\phi, \kappa_{OA}, \psi_A) \right]^2 d\phi. \quad (10)$$

The adopted criterion is important, yet still insufficient. One has to bear in mind that it is crucial that the derivative $dy/d\phi$ be minimised, since it determines the inertia forces acting upon the slewing mechanism during the hoisting or lowering the jib. With the weight R being ascribed to the function Δy , and the function $dy/d\phi$ added with its ascribed weight P , we get a new optimisation criterion which combines the two previous criteria [1].

$$J(\kappa_{OA}, \psi_A) = \int_{\phi_{\min}}^{\phi_{\max}} \left\{ P \cdot \left[\frac{dy(\phi, \kappa_{OA}, \psi_A)}{d\phi} \right]^2 + R \cdot \left[\Delta y(\phi, \kappa_{OA}, \psi_A) \right]^2 \right\} d\phi. \quad (11)$$

Restrictions imposed on the fixed jib length L_{OB} :

$$\left\{ \begin{array}{l} i_w \geq 3 \text{ where: } i_w = 2n+1, \text{ and } n=1,2,\dots, \\ \frac{\pi}{3} \leq \Psi_A \leq \frac{2\pi}{3}, \\ 0 \leq k_A \leq \frac{1}{2}. \end{array} \right. \quad (12)$$

When the functional (10) is replaced by (11) in the optimisation problem 1 for the stable conditions (12), we get new solutions for the parameters κ_{OA}, Ψ_A . It appears, that extending the optimisation criterion to incorporate the condition imposed upon the derivative $dy / d\varphi$ leads to the minimisation of the inertia force and further, reduces the deviation from the straight-line trajectory in the payload's motion in relation to solution (10) [1, 2, 3]. For two values of transmission of the compensating rope system: $i_w = 3$ and $i_w = 5$ for the fixed values of weight coefficients $P = 1$ and $R = 1$ in functional (12), we get:

$$i_w = 3 \rightarrow \left\{ \begin{array}{l} \kappa_{OA} = 0.3078 \\ \Psi_A = 83.2674^\circ \end{array} \right., \quad i_w = 5 \rightarrow \left\{ \begin{array}{l} \kappa_{OA} = 0.1901 \\ \Psi_A = 85.9712^\circ \end{array} \right. \quad (13)$$

The hook's trajectory deviation from the straight-line depends on the transmission ratio of lifting mechanism and becomes for $i_w = 3 \rightarrow \delta_3 = 1.242\%$, and for $i_w = 5 \rightarrow \delta = 0.666\%$.

Fig. 3 shows the trajectory of the hook when the crane radius changes over its entire range, for each pair of solutions (13). When the transmission ratio of the jib lifting mechanism is increased, the payload's horizontal trajectory better approximates the straight-line motion; however, the rope resistance due to winding is also increased.

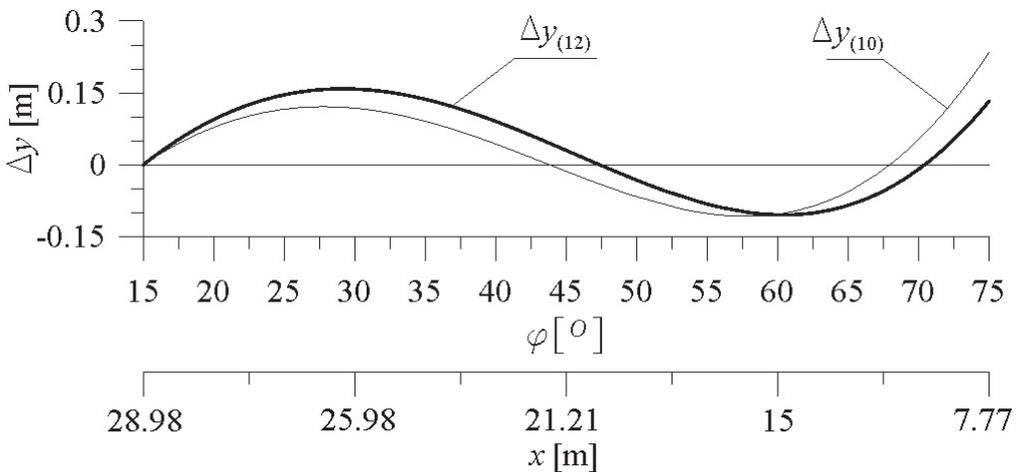


Fig. 3. Hook's trajectory during the radius change over its full range obtained for two criterions: (10) and (12)

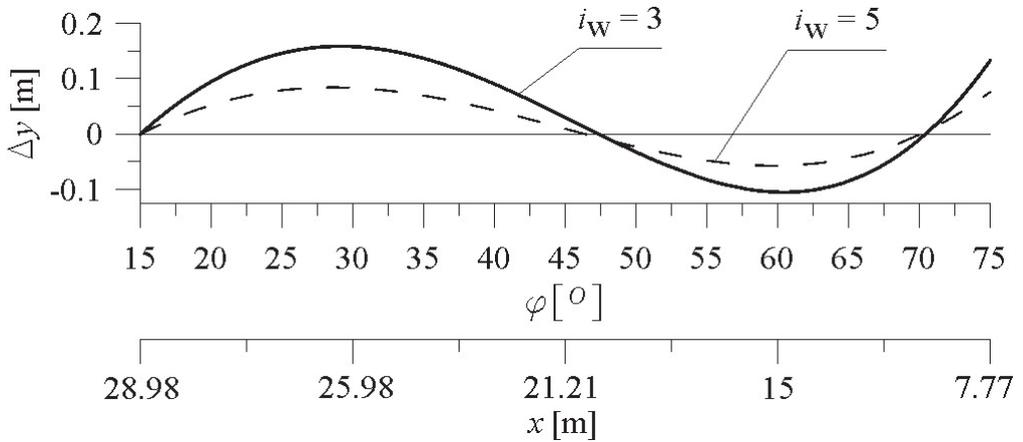


Fig. 4. Hook's trajectory during the radius change over its full range for two values of the transmission ratio of lifting mechanism $i_w = 3$ and $i_w = 5$

3. Parametric optimisation of the jib balance system

Balancing of the jib in a one-link crane requires the selection of the jib ballast weight and position of the pulley, with respect to the jib's rotation axis, in such a manner as to minimise the work required for slewing change. Figure 1 shows the loads acting on the jib in a one-link crane. Recalling the previous optimisation problem, the following designations are adopted: L_{OG} – distance between the rotation axis of the jib O and the pulley G , ψ_G – angle of horizontal inclination of the line segment \overline{OG} . The residual unbalanced moment of the jib is a function of the angular position ϕ :

$$M(\phi) = (G_w L_{OS} + QL_{OB}) \cos(\phi) - G_P L_{OF} \frac{L_{OG} \sin(\psi_G - \phi)}{L_{GF}(\phi)} - 3QL_{OB} \frac{L_{OA} \sin(\psi_A - \phi)}{L_{AB}(\phi)}. \quad (14)$$

$$\text{where: } L_{GF}(\phi) = \sqrt{L_{OF}^2 + L_{OG}^2 - 2L_{OF}L_{OG} \cdot \cos(\psi_G - \phi)}. \quad (15)$$

Optimisation problem 2

In this optimisation task, $M(\phi, L_{OG}, \psi_G, L_{OP}, G_P)$, becomes the objective function and the decision variables are: position of the pulley G defined by distance – L_{OG} and angle – ψ_G , distance from the axis of the pin jib – O to the point where the rope counterweight is attached to the arm of – L_{OP} , weight of counterweight – G_P . Distances: L_{OA} , L_{OB} , L_{OS} and weight Q and G_w are constant parameters in the optimisation procedure. For counterweight configurations as shown in Fig. 1, where the rope is connected to the jib creating a mechanism with unilateral bonds, the jib is balanced without the payload Q . The task consists of the determination of the optimal values: $L_{OG}, L_{OP}, \psi_G, G_P$ for which the quadratic functional $J(L_{OG}, L_{OP}, \psi_G, G_P)$, reaches a minimum.

$$J(L_{OG}, \psi_G, L_{OF}, G_P) = \int_{\phi_{\min}}^{\phi_{\max}} [M(\phi, L_{OG}, \psi_G, L_{OF}, G_P)]^2 d\phi. \quad (16)$$

$$\text{Assumed condition: } \left\{ \begin{array}{l} \psi_A, L_{OA}, L_{OB}, L_{OS}, G_W, Q (Q=0) - \text{known parameters,} \\ M(\phi, L_{OG}, \psi_G, L_{OF}, G_P) > 0 \text{ for each } \phi_{\min} \leq \phi \leq \phi_{\max}, \\ 0.5G_W \leq G_P \leq 1.7G_W \\ 0 < L_{OG} \leq \frac{1}{2}L_{OB} \\ 0 < L_{OF} \leq L_{OB} \\ \frac{\pi}{3} \leq \psi_G \leq \frac{2}{3}\pi \end{array} \right. \quad (17)$$

The solution to the optimisation task No. 2, for $G_w = 45 \text{ kN}$, $L_{OS} = 12.857 \text{ m}$, $L_{OA} = 9.234 \text{ m}$, $\psi_A = 83.2674^\circ$, is the set of parameter values such that the functional (16) is minimised for the imposed constraint conditions (18):

$$\left. \begin{array}{l} L_{OG} = 7.0605 \text{ m} \\ L_{OF} = 30 \text{ m} \\ \psi_G = 85.489^\circ \\ G_P = 67 \text{ kN} \end{array} \right\} \rightarrow L_\phi = \int_{\phi_{\min}}^{\phi_{\max}} M(\phi, L_{OG}, \psi_G, L_{OF}, G_P) d\phi = 51.43 \text{ kJ} \quad (18)$$

It appears that the best solution is obtained when the counterbalance rope is attached to the end of the jib, i.e. when $L_{OF} = L_{OB}$. Optimisation of the rope mechanism in the counterweight rope is discussed in more detail in [1] and the assumption that the counterbalance rope is attached to the tip of the jib is adopted based on a review of the existing crane design options. From the standpoint of mathematics, solution (18) confirms the validity of this assumption. Polar coordinates of the pulley mechanism in a counterweight are $-G [7.0605 \text{ m}, 85.489^\circ]$.

The effectiveness of the parametric optimisation of the movable counterweight mechanism is expressed as work $-L_\phi$ needed to change the jib's angular position over the entire variability range of its inclination angle.

Under conditions defined thus, this quantity is expressed as the integral (18) and equals 51.43 kJ. Finally, it is recommended that the counter jib ballast weight should be taken as 10% less than the value predicted in the optimisation problem to make up for resistance due to friction when the jib is lowered. This recommendation applies only to unilaterally constrained mechanisms. When a mechanism with bilateral constraints is considered, for example, a lever mechanism in a four-bar linkage, the positive residual torque requirement can be eliminated. The jib slewing work could be effectively reduced through optimising the structural design of such a mechanism.

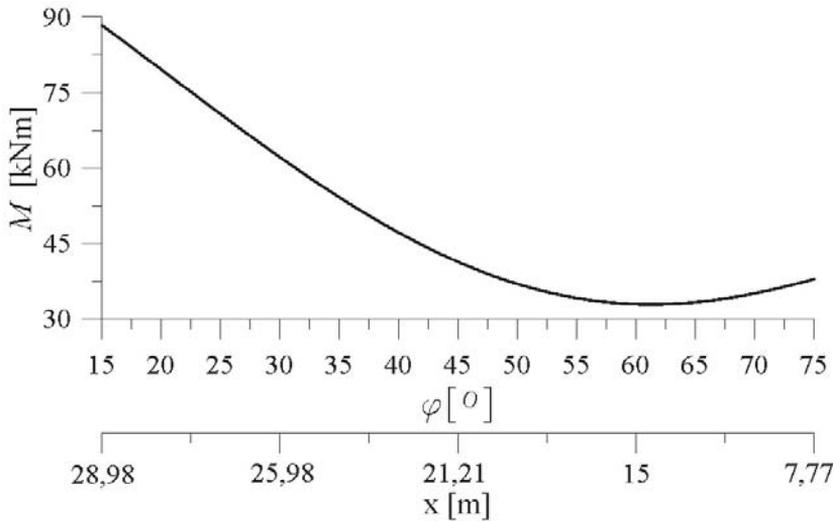


Fig. 5. Residual moment of jib unbalance as a function of the radius change

4. Parametric optimisation of the jib lifting mechanism

Optimisation of the slewing mechanism discussed in Section 1 could in fact be applied to the rope mechanism in a winch. The main objective was to ensure such roping configuration so as to minimise the horizontal hook trajectory error for the full variability range of the change in the jib's angle of horizontal inclination when the winch is blocked. Forces required to lift the jib have not been considered so far. Recalling (14), the force acting on the rope lifting the jib can be written as:

$$S_w(\phi) = \frac{L_{EW}(\phi)}{L_{OE}L_{OW}\sin(\psi_w - \phi)} M(\phi), \quad (19)$$

$$\text{where: } L_{EW}(\phi) = \sqrt{L_{OW}^2 + L_{OE}^2 - 2L_{OW}L_{OE} \cdot \cos(\psi_w - \phi)}. \quad (20)$$

Optimisation problem 3

The optimisation problem involving the jib lifting mechanism consists of finding the point where the rope is attached to the jib, as well as the polar coordinates of the pulley axes – W [L_{OW} , ψ_w] associated with the tower crane, for which the force in the winch will be the lowest in terms of rms value and should be positive. All parameters determined in earlier sections remain constant throughout the optimisation of lifting the jib mechanism.

$$J(L_{OE}, L_{OW}, \psi_w) = \int_{\phi_{\min}}^{\phi_{\max}} [S_w(\phi, L_{OE}, L_{OW}, \psi_w)]^2 d\phi, \quad (21)$$

$$\text{Assumed condition: } \left\{ \begin{array}{l} \psi_A, \psi_G, L_{OA}, L_{OB}, L_{OS}, L_{OA}, L_{OB}, G_w, Q, G_p - \text{known parameters,} \\ S_W(\phi, L_{OE}, L_{OW}, \psi_W) > 0 \text{ for each } \phi_{\min} \leq \phi \leq \phi_{\max}, \\ 0 < L_{OE} \leq L_{OB} \\ 0 < L_{OW} \leq \frac{1}{3} L_{OB} \\ \frac{\pi}{3} \leq \psi_W \leq \pi \end{array} \right. \quad (22)$$

The solution to the optimisation task No. 3 for the nominal value of load $Q = 50$ kN is the following set of parameter values that minimise functional (21), under the imposed conditions (22):

$$\left. \begin{array}{l} L_{OE} = 30m \\ L_{OW} = 10m \\ \psi_W = 116.4911^\circ \end{array} \right\} \quad (23)$$

The best solution is obtained when the rope is attached to the end of the jib, that is when $L_{OE} = L_{OB}$. Polar coordinates of the pulley axes W [10 m, 116.4911°]. The torque required to hoist the jib expressed in terms of (14) is not dependent upon parameters of the jib lifting mechanism and neither is the work required to lift the jib.

For previously determined parameters of the slewing and counterweight mechanisms, and under the loading conditions due to the nominal payload – Q and the weight of the jib G_w operating at the distance – L_{OS} from the axis of the pin jib, the hoisting work becomes:

$$L_\phi = \int_{\phi_{\min}}^{\phi_{\max}} M(\phi) d\phi = 58.07 \text{ kJ} \quad (24)$$

Optimisation of the jib lifting mechanism results in the reduction and balancing of forces in the rope over the entire range of angle variability $\phi \in [15-75^\circ]$.

In Fig. 6, the forces are compared that act on the rope hooked on the movable end of the jib and which runs through a pulley located at the point A or G or W , depending on the design option, in accordance with designations shown in Fig. 1. Three plots of force acting on the jib winch rope are derived and their common feature is the minimum value achieved for the slewing angle – $\varphi \approx 50^\circ$. The least favourable force variability pattern is obtained when the axis of the pulley in the jib lifting mechanism coincides with that in the pulley of counterweight mechanism – G . The values of force acting in the rope – S_w decrease from 22.1 kN to nearly zero then rise again to achieve the maximum angle of deception, which is 84.0 kN (dashed line – G in Fig. 6). The variability pattern of the force acting on the rope was achieved when the axis of the pulley in the jib lifting mechanism coincides with that of the pulley in the winch mechanism – A . The force value S_w decreases from 16.85 kN to nearly zero and then rises again to 73.6 kN for a maximum value of angle φ , (thin line – A in fig. 6).

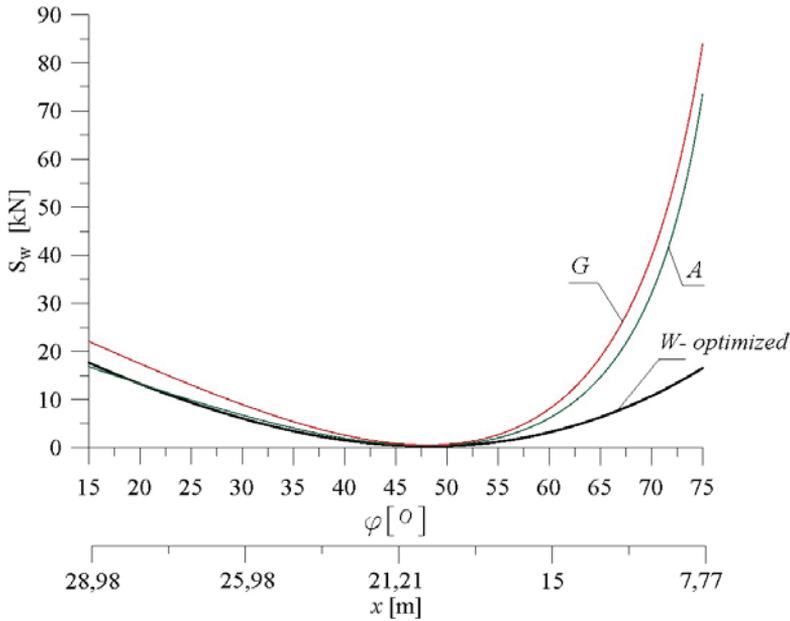


Fig. 6. Controlling forces acting in the rope through selection of the rotation axis of the pulley in the jib-lifting mechanism (plot designations correspond to points A, G, W in Fig. 1)

The most favourable pattern of force acting upon the rope is obtained when the axis of rotation of the pulley in the jib lifting mechanism is at point – W. The values of force S_w go down from 17.67 kN to nearly zero and then rises again to approaching 16.56 kN for the maximum value of the angle φ (thick line – W in Fig. 6).

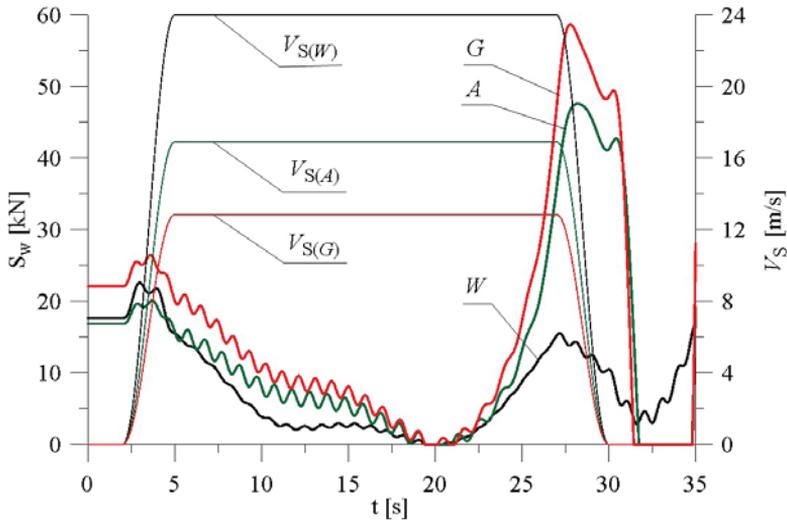


Fig. 7. Comparison of dynamic forces acting in the rope depending on the position of the pulley in the jib-lifting mechanism (plot designations correspond to points A, G, W in Fig. 1)

To verify the optimization results in dynamic conditions it is required that flexibility in the system, including rope flexibility, should be taken into account. Basing on the catalogue data, the universal model of kinematic excitations is adopted differing in the steady-state velocity values for specific mechanisms. The duty cycle of each mechanism involves the following stages: start, steady-state motion and braking, yielding a trapezoidal characteristics of drive velocity. Figure 7 confirms the optimization results under dynamic conditions.

Advantages of minimising the force acting on the rope in the jib lifting mechanism are:

- ▶ Small rope diameter → small pulley → low resistance during rope winding
- ▶ Low-power electric motors (approximately 7 kW) → reduced energy demand
- ▶ Small force variations in ropes → less overloading of electric motors → little overheating of engines

Because of the unilateral constraints in rope mechanisms, it is recommended in the optimisation process that the jib weight should be taken 10% less than in real life conditions.

5. Concluding remarks

Optimisation tasks involving the three-rope mechanisms in a one-link jib crane lead us to the following conclusions:

- 1) The application of dedicated software (such as Mathcad) to solve variational problems such as finding a minimum of properly formulated quadratic functionals proves to be a very effective and rapid solution to parametric optimisation problems.
- 2) Even though functionals (9), (10), (14), (19) are formally quadratic, it is not required that the Riccati equations be solved.
- 3) When the optimisation criterion for the slewing mechanism is extended to incorporate the condition imposed on the derivative $dy/d\varphi$, the form of the quadratic functional (9) becomes more complicated, but the numerical solution can still be found.
- 4) The optimisation effectiveness of the boom luffing mechanism determines the level of vibration reduction of the cargo hung on the hook.
- 5) The optimisation problem is solved and solutions are obtained in the form of a set of mechanism parameters for which the work involved in payload hoisting should be minimal. The force acting in line in the jib lifting mechanism should be minimal.
- 6) For the assumed lifting capacity and distance jaunt, we get a crane mechanism structure that guarantees the minimal energy consumption.

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