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MODELLING OF FUZZY LOGIC CONTROL SYSTEM USING THE MATLAB SIMULINK PROGRAM

MODELOWANIE UKŁADU STEROWANIA W LOGICE ROZMYTEJ PRZY UŻYCIU PAKIETU MATLAB SIMULINK

A b s t r a c t

The main aim of this work was building model of fuzzy logic controller for dynamic system on example of inverted pendulum. This problem was solved using Matlab 2008 environment with Simulink module and Fuzzy Logic Toolbox. Obtained results of simulations show that regulator based on fuzzy logic can be efficiently used for controlling non linear dynamic system.

Keywords: control, fuzzy logic, inverted pendulum, simulation, Matlab, Simulink

S t r e s z c z e n i e

Celem pracy było zbudowanie modelu regulatora w logice rozmytej dla układu dynamicznego na przykładzie odwróconego wahadła. Postawione zadanie zostało wykonane przy wykorzystaniu środowiska Matlab 2008 z modułami Simulink oraz Fuzzy Logic Toolbox. Uzyskane wyniki symulacji pokazują, że układ w logice rozmytej może zostać efektywnie wykorzystany do sterowania nieliniowym układem dynamicznym.

Słowa kluczowe: sterowanie, logika rozmyta, odwrócone wahadło, symulacja, Matlab, Simulink

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1. Introduction

Nowadays, in research centers all over the world are undertaken efforts in order to design control systems for more and more complex devices and processes. In connection with this fact new control methods and algorithms are required. Especially significant are studies on fuzzy logic control systems [1]. Fuzzy logic controllers are designed particularly for non-linear dynamic systems with many inputs and outputs which are so complex, that it is very difficult or even impossible to build exact mathematical model. Controllers of this type are characterized by large number of parameters, like: number and form of fuzzy sets used for division of computational domain of input and output signals, selection of operators for realizing fuzzy operations as sum, product and negation, selection of conclusion algorithm, choice of function for computing numerical values of output signals [5]. These parameters must be adjusted and given the correct values in order to the controller work properly. The main aim of work presented in this paper was to analyze, create models and carry out simulations of fuzzy logic controller for non-linear dynamic system on the example of inverted pendulum [4, 7]. Model of both inverted pendulum and fuzzy logic controller were created in Matlab – Simulink system.

2. Application of Fuzzy Logic in control systems

Application of fuzzy logic in control process requires using the following elements: definitions of fuzzy sets, fuzzy logic operators, fuzzy rules (rule database), inference mechanism and algorithm of computing numerical values of output signal.

2.1. Definitions of fuzzy sets

The following kinds of fuzzy sets were used in model of fuzzy logic controller: trapezoidal, built of line segments and defined by continuous functions: Gauss-shaped and bell-shaped. Trapezoidal function defined at range $\langle a \quad b\rangle$ with nucleus $\langle c \quad d\rangle$ can be defined using formula (1):

$$\mu_F(x) = \begin{cases} 0, & \text{for } (x<a)\cup(x>b) \\ \dfrac{x-a}{b-a} & \text{for } a \le x < b \\ 1 & \text{for } b \le x < c \\ \dfrac{d-x}{d-c} & \text{for } c \le x \le d \end{cases} \tag{1}$$

Equations (2) and (3) describe Gauss and bell-shaped functions, respectively:

$$\mu_F(x) = e^{-(x-C_F)/w}, \tag{2}$$

$$\mu_F(x) = \frac{1}{1+(x-C_F)^2}, \tag{3}$$

where:

C_F – to nucleus of fuzzy set μ: $\text{nuc}(F) = \{x \in X : \mu(x) = 1\}$,

w – parameter.

2.2. Fuzzy relational operators

Several different versions of fuzzy sum and product operators can be used in model of fuzzy regulator [5]. The most commonly used fuzzy sum operators are MAX or SUM, while product operators are MIN or PROD. Operators of sum of two fuzzy sets A and B can be described using formulas (4) and (5). Notations of product operators of the same fuzzy sets are presented using formulas (6) and (7):

$$\mu_{A \cup B}(x) = \text{MAX}(\mu_A(x), \mu_B(x)), \quad (4)$$

$$\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), \quad (5)$$

$$\mu_{A \cap B}(x) = MIN(\mu_A(x), \mu_B(x)), \quad (6)$$

$$\mu_{A \cap B}(x) = \mu_A(x) \cdot \mu_B(x), \quad (7)$$

2.3. Construction of rule database

A rule can be formulated using fuzzy operators in IF – THEN statement. The general form of the fuzzy rule is:

$$IF\ R_i\ \text{THEN}\ u_j = f_j(x_1, x_2, \ldots, x_n), \quad (8)$$

where:

R_i – relation between fuzzy sets of input parameters defined using fuzzy operators,

U_j – output signal,

F_j – function,

$x_1 \ldots x_n$ – numerical values of input signals.

According to Takagi – Sugeno model fi is usually a polynomial function. If a function is reduced to constant value, a zero-order fuzzy controller is obtained. This type of controller is called a Mamdani model. In this case fuzzy rule can be formulated as:

$$IF\ R_i\ \text{THEN}\ u_j = A_{jk}, \quad (9)$$

where:

A_{jk} – singular fuzzy set of output value j.

2.4. Methods of generating output signals

Each fuzzy rule describes influence of individual fuzzy set of input signals on the individual fuzzy set of output signal. Numerical procedure which allows for determining this influence is called fuzzy implication. The implication can be defined in many ways [3] but the most common use is previously mentioned product operators MIN or PROD [2]. The influences produced by all rules applying to individual fuzzy set of output signal should be then combined together in order to obtain the total assessment of this signal. This process is called fuzzy inference. The result of inference process is a fuzzy set of output signal. In practice, each controller should generate a non-fuzzy (crisp) numerical output signal. Procedure which generates numerical value from fuzzy set of output signal is known as defuzzification process. The output signal can be calculated using several methods [6]. One of the most common is CoS (Center of Sum). It uses the following formula [2]:

$$U(x_1 \ldots x_n) = \sum_{i=1}^{n} u_i \cdot \mu_u(x_1 \ldots x_n, u_i) \Big/ \sum_{i=1}^{n} \mu_u(x_1 \ldots x_n, u_i)\,, \tag{10}$$

where:

- U – crisp value of output signal,
- n – number of fuzzy sets of output signal,
- u_i – individual element of fuzzy set of output signal,
- μ_u – membership function of element u_i.

Output signal can be also calculated using the COG method (Center of Gravity). For discrete computational domain this method can be written as [2]:

$$U(x_1 \ldots x_n) = \sum_{i=1}^{n} u_i \cdot \left(\sum_{j=1}^{m} \mu_{Rj}(x_1 \ldots x_n, u_i) \right) \Big/ \sum_{i=1}^{n} \sum_{j=1}^{m} \mu_{Rj}(x_1 \ldots x_n, u_i)\,, \tag{11}$$

where:

- m – number of fuzzy rules activated by inputs $x_1 \ldots x_n$.

Continuous computational domain requires exchange of sum operators in equations (10) and (11) to integrals.

3. Model of the system

Considered system consists of mathematical model of inverted pendulum and model of fuzzy logic controller. Model of the pendulum was created in Matlab – Simulink program, while fuzzy logic controller was built using Matlab Fuzzy Logic Toolbox. Simulations were carried out in Simulink.

3.1. Mathematical model of inverted pendulum

Application of fuzzy logic controller will be shown on example of inverted pendulum system. It is commonly known, that inverted pendulum is inherently unstable and has strongly non-linear properties [4, 7], so it is difficult to control. Diagram of considered pendulum is shown in Fig. 1.

a) b)

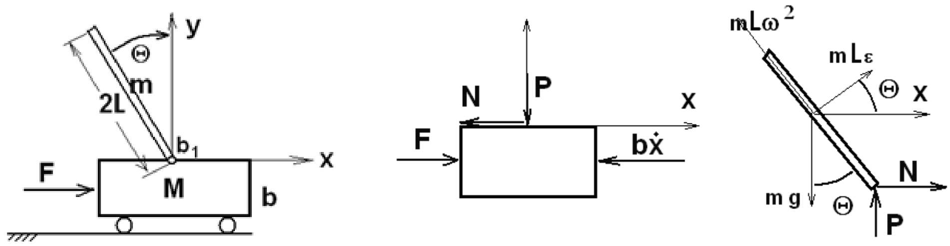


Fig. 1. Diagram of inverted pendulum a) system, b) subsystems of frame and pendulum

Rys. 1. Schemat poglądowy odwróconego wahadła: a) układ, b) podukłady wózka i wahadła

Parameters defined in fig. 1 are as follows: x – horizontal position of pendulum frame [m], $\dot{x}$ - horizontal velocity of pendulum frame [m/s], Θ – rotation angle of pendulum [rad], $\dot{\Theta} = \omega$ – angular velocity, $\dot{\omega} = \varepsilon$ – angular acceleration, *M*, *m* – mass of frame and pendulum, *b*, b_1 – coefficient of viscous friction of frame and pendulum [Ns/m], I - moment of inertia $I = 4/3 \cdot \mathrm{m} \cdot L^2$ [kg m^2], 2*L* – length of pendulum [m], *N*, *P* – horizontal and vertical component of reactive force [N], *g* – acceleration of gravity: 9.81 [m/s^2], *F* - control force [N].
Mathematical model of this system is made up of four equations: sum of forces acting on frame projected on horizontal axis, sum of forces acting on pendulum projected on horizontal axis, sum of forces acting on pendulum projected on axis perpendicular to it and sum of moments acting on pendulum around its center:

$$\begin{cases} M \cdot \ddot{x} = F - N - b \cdot \dot{x} \\ m \cdot \ddot{x} = N + m \cdot L \cdot \omega^2 \cdot \sin(\Theta) - m \cdot L \cdot \varepsilon \cdot \cos(\Theta) \\ m \cdot \ddot{x} \cdot \cos(\Theta) + m \cdot L \cdot \varepsilon = P \cdot \sin(\Theta) + \cdot N \cdot \cos(\Theta) - \cdot m \cdot g \cdot \sin(\Theta) \\ I \cdot \varepsilon = -b_1 \cdot \omega - P \cdot L \cdot \sin(\Theta) - N \cdot L \cdot \cos(\Theta) \end{cases}, \tag{12}$$

Presented equations can be easily converted by eliminating *N* and *P* parameters. Final system consists of two equations:

$$\begin{cases} (M + m) \cdot \ddot{x} + b \cdot \dot{x} - m \cdot L \cdot \omega^2 \cdot \sin(\Theta) + m \cdot L \cdot \varepsilon \cdot \cos(\Theta) = F \\ (I + m \cdot L^2) \cdot \varepsilon + b_1 \cdot \omega + m \cdot g \cdot L \cdot \sin(\Theta) = -m \cdot L \cdot \ddot{x} \cdot \cos(\Theta) \end{cases}, \tag{13}$$

Equations (13) were used to create model in Simulink. The model is presented in Fig. 2.

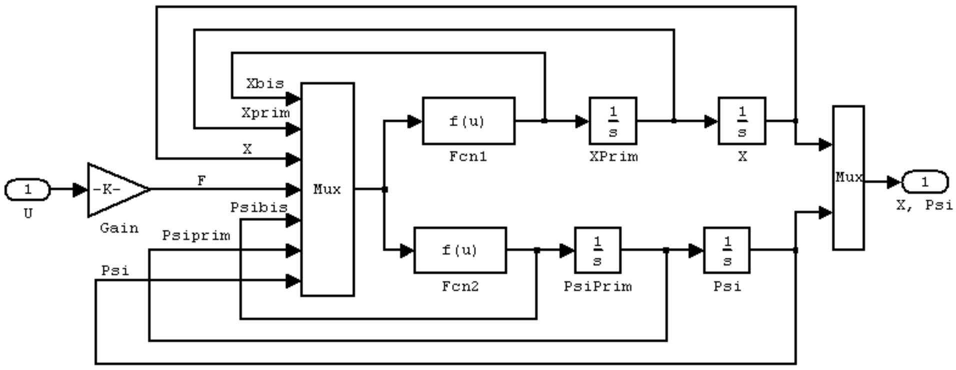


Fig. 2. Model of inverted pendulum created in Simulink

Rys. 2. Model odwróconego wahadła zbudowany w Simulinku

3.2. Model of fuzzy logic controller

To control inverted pendulum a mamdani type fuzzy logic controller was used. The structure of the controller in Fuzzy Logic Toolbox window is presented in Fig. 3.

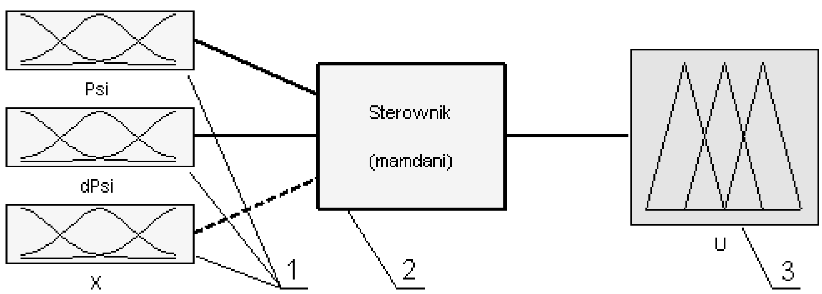


Fig. 3. Structure of fuzzy logic controller in Fuzzy Logic Toolbox window: 1) input signals, 2) rule database and inference algorithm, 3) output signal

Rys. 3. Struktura regulatora rozmytego w oknie pakietu Fuzzy Logic Toolbox: 1) sygnały wejściowe, 2) baza reguł i algorytm wnioskujący, 3) sygnał wyjściowy

As it comes from the figure, controller has three input signals: rotation angle of pendulum called *Psi*, rotation velocity *dPsi* and position of pendulum frame *X*. Output signal *U* is proportional to force acted at pendulum frame.

3.3. Control process strategy

Controller was applied in a regulation system using feedback of rotation angle value and pendulum frame position. Regulation system is shown in Fig. 4. The system consists of the following blocks: required angle of inverted pendulum equal to π rad, subsystem which computes input values for the controller Inputs, fuzzy logic controller FLC, saturation block keeping the control signal within given bounds, switch which allows for carrying out simulation without controller and subsystem of pendulum. Values of control signal, rotation angle of pendulum and position of pendulum frame are presented in graphical form as time courses and saved into file using the Results subsystem.

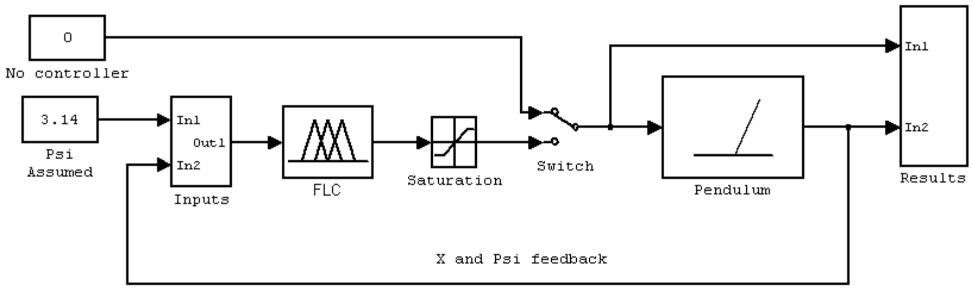


Fig. 4. Regulator of inverted pendulum with application of fuzzy logic

Rys. 4. Układ regulacji odwróconego wahadła z zastosowaniem logiki rozmytej

4. Simulation tests and results

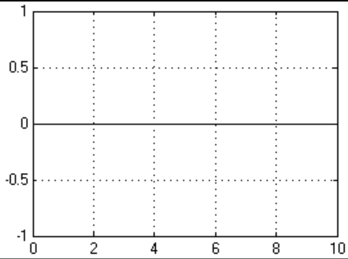
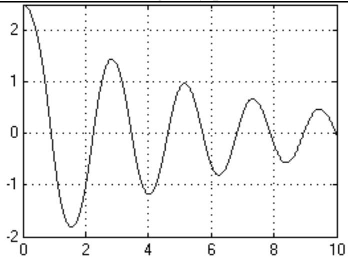
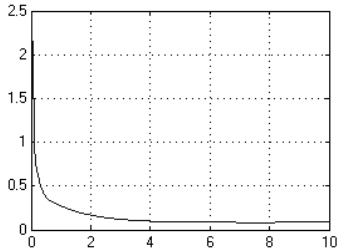
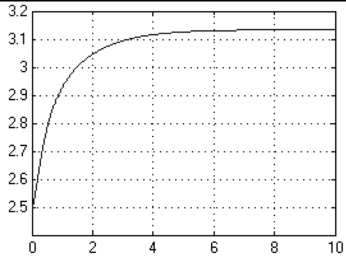
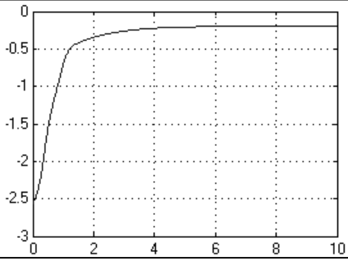
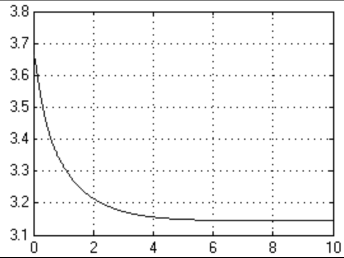
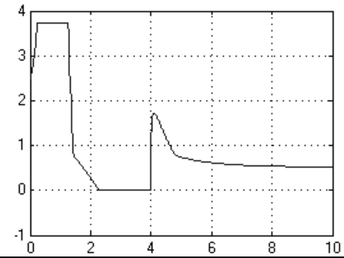
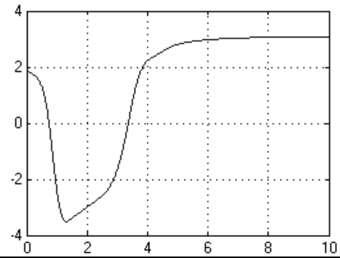
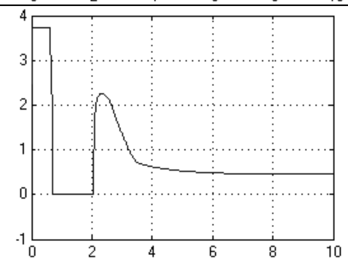
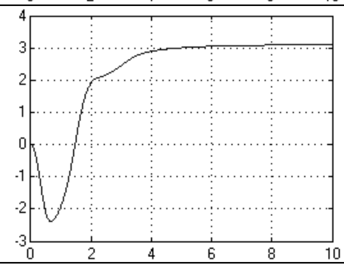
The following values of physical system parameters were assigned: $M = 0.9\,\text{kg}$, $m = 0.3\ \text{kg}$, $L = 0.5\,\text{m}$, $b = 0.1\,\text{Ns/m}$, $b_1 = 0.05\,\text{Ns/m}$. Simulation tests were carried out with various initial conditions of rotation angle $\Theta(0)$ and the other initial conditions set to zero: $\dot{\Theta}(0) = x(0) = \dot{x}(0) = 0$.

Example simulation results are shown in table 1.

In case a), when no controller is applied, rotation angle oscillated around zero. It means that pendulum was reaching vertically down position. Fuzzy logic controller was applied in cases b) to e) in order to obtain vertically up position of pendulum, regardless the initial position. In cases b) and c) at starting point of simulation the pendulum is got out of plumb by 30 degrees in plus and in minus. It comes from the results, that in both cases fuzzy logic controller allowed for obtaining desired position of pendulum. In case d) starting angle of pendulum was increased to 75 degrees in minus. Despite the control signal, pendulum goes down, crosses the zero angle, deflects in the opposite direction. Then it goes back, finally reaching the required position. The similar situation appears in case e), when starting position of pendulum is vertically down. At first pendulum deflects in the opposite direction, but fuzzy logic controller allows for obtaining the assumed position.

Table 1

Example results of simulations

Params	*Control signal*	*Rotation angle of pendulum*
a) $\Theta(0) =$ 2.617 rad (-300 out of plumb) No control		
b) $\Theta(0) =$ 2.617 rad (-30^0 out of plumb) FLC controller		
c) $\Theta(0) =$ 3.664 rad (+300 out of plumb) FLC controller		
d) $\Theta(0) =$ 1.831rad (-750 out of plumb) FLC controller		
e) $\Theta(0) = 0$ (vertically down) FLC controller		

5. Conclusions

The objective of this work was building model of fuzzy logic controller for non linear dynamic system on example of inverted pendulum system. Model of the inverted pendulum system was built in Matlab – Simulink. Model of fuzzy logic controller was designed using the Fuzzy Logic Toolbox. Results of simulations confirm, that regardless of initial position of pendulum, fuzzy logic controller is always capable for obtaining desired vertically up position.

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