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NEW CONCEPT OF FINITE ELEMENT METHOD FOR FGM MATERIALS

NOWA KONCEPCJA METODY ELEMENTÓW SKOŃCZONYCH DLA MATERIAŁÓW FGM

Streszczenie

Celem niniejszego artykułu jest prezentacja nowego typu niejednorodnego elementu skończonego służącego do modelowania cienkiej warstwy materiału kompozytowego FGM. Cechą główną materiału FGM (functionally graded materials) jest gładka, funkcyjna zmienność parametrów materiałowych, takich jak moduł Younga, współczynnik rozszerzalności termicznej czy współczynnik przewodności termicznej w zależności od zmiany mikrostruktury. Materiał kompozytowy zawierający metal lub stop metali jako matrycę i materiał ceramiczny jako włókna bez warstwy przejściowej FGM narażony jest na powstawanie lokalnych koncentracji naprężeń na granicach interfejsu. Klasycznie sformułowany element skończony zawiera stałe parametry materiałowe, co prowadzi do znacznych błędów numerycznych. Rozwiązaniem tego problemu jest zastosowanie sformułowania elementu skończonego z dodatkową funkcją aproksymacyjną służącą do interpolacji własności materiałowych na poziomie każdego elementu. W praktyce materiałowe funkcje kształtu są definiowane jako funkcje eksponencjalne lub potęgowe opisujące indywidualny charakter niejednorodności.

Słowa kluczowe: FGM, niejednorodny MES, termosprężystość, cienka warstwa

Abstract

The aim of this paper is new concept of nonhomogeneous finite elements due to FGM composite material. The main idea of functionally graded materials is a smooth variation of material properties, such as modulus of elasticity, coefficient of thermal conductivity or coefficient of thermal expansion, due to continuous change of microstructure. Composite material containing metal or metal alloy as a matrix and ceramic as fibres without FGM interface thin layer may lead to damage or failure due to delamination of the ceramic film from the substrate. This is the result of localized stress gradients at the interface. Classical finite element formulation contains constant material properties, which leads to numerical errors. A proper approach to solve this problem requires application of nonhomogeneous FE containing additional approximation functions in order to interpolate material properties at the level of each finite element. In practice, material shape functions can be represented by exponential or power functions describing individual character of inhomogeneity.

Keywords: FGM, nonhomogeneous FEM, thermoelastic, thin layer

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1. Introduction

In last few years, material processing allowed to manufacture special kind of the material composite called functionally graded materials (FGM). Thermomechanical properties of those materials varied along the thickness. In this case, FGM materials are nonhomogeneous [1–4].

Single-component materials are usually characterized either by high thermal resistance like ceramics or high strength, like metals. Composite materials contain two or more constituents (phase), where the ceramic phase is usually characterized by higher thermal and corrosion resistance. Metal or metal alloy phase is characterized by high strength and toughness. However, discontinuity of thermomechanical properties at the coating substrate interface causes stress concentrations and consequently, transverse cracks or delamination. Those kind of failures follow loss of functionality due to locally removed coating by spalling. As a remedy to this rupture, in 1980 Japanese engineering and scientists involved new generation of ceramic materials, which contains interface with varying, graded material properties [2–4].

2. Motivation

Functionally graded composites, with smooth variation of volume fractions, offer various advantages, such as reduction of residual stress [5] and increased bonding strength [6]. Moreover, if properly used, such materials also lead to reduction of stress concentration or stress intensity factor [7]. For example, Hasselman and Youngblood [8] found that the maximum tensile thermal stresses in brittle ceramic can be reduced significantly by spatially varying thermal conductivity in hollow circular cylinder subjected to radially inward and outward steady-state heat flow.

The thickness of the FG interface is usually small. Moreover the changes of thermomechanical properties are huge. For example, the thermal conductivity for aluminium alloy is equal 154 [W/mK] in comparison to 2 [W/mK] for ceramic material $ZrO2 + Y2O3$, after Wang [9] and Lee [10].

Classical finite element formulation contains constant constitutive matrix on the FE level. This assumption leads to errors. Numerical simulations of the structure made of functionally graded materials are causing convergence problems. More information and further discussion about classical finite element method in comparison to another numerical method is possible to find in [11–12].

The aim of this paper is an advanced formulation of finite element method for nonhomogeneous material. The advantages of nonhomogeneous FEM will be presented based on numerical example.

3. Finite element formulation

3.1 Classical homogeneity isoparametric finite element

Isoparametric elements are characterized by the shape function, used to approximate the geometry of the element and the unknown function. In this case the displacement function and definition of the coordinate can be written as

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$$u^e = \sum_{i=1}^{m} N_i u_i^e, \quad x = \sum_{i=1}^{m} N_i x_i \tag{1}$$

where:

N_i – is a shape function, $\boldsymbol{u}_i$ is the nodal displacement corresponding to i-th node and m is the number of nodal points in the element.

For example, for a Q4 element, the standard shape function is given as follows

$$N_i = \frac{1}{4}\left(1+\xi\xi_i\right)\left(1+\eta\eta_i\right), \quad i = 1,\cdots,4 \tag{2}$$

where:

$\left(\xi,\eta\right)$ – denotes intrinsic coordinates in the interval [–1, 1] and $\left(\xi_i,\eta_i\right)$ denotes the local coordinates of node i. Usually, strains are obtained from displacement by differentiation as

$$\varepsilon^e = B^e u^e \tag{3}$$

where:

B^e – is the strain-displacement matrix of shape function derivatives, and u^e is the nodal displacement vector. Thus strain-stress relations are given by

$$\sigma^e = D^e \varepsilon^e \tag{4}$$

where:

D^e – is the constitutive matrix, and σ^e is the stress tensor. The principle of virtual work yields the following finite element stiffness equations

$$k^e u^e = F^e \tag{5}$$

where:

F^e – is the load vector and the element stiffness matrix is given by integral

$$k^e = \int_{\Omega^e} \left(B^e\right)^T D^e B^e \mathrm{d}\Omega^e \tag{6}$$

in which Ω^e is the domain of element (e), and T denotes transpose. The reasoning above, at the element level, can be readily extended to whole domain, which leads to a system of algebraic equations for the unknown nodal displacements.

It is worth to noticed that in classical finite element formulation the constitutive matrix is constant. The material properties vary in a piecewise continuous manner, from one element, to the other, but all integration points within an element have a common property value. It is possible to shift the constitutive matrix in front of the integral in Eq. (6).

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3.2. Isoparametric nonhomogeneous finite element formulation

For simplicity of notation, the superscript (e), connecting to element, is dropped in the further part of this paper.

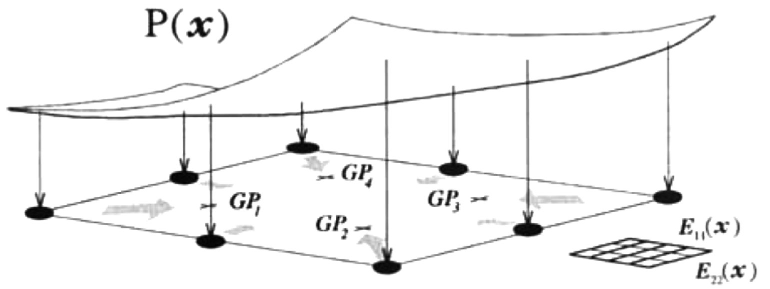

Fig. 1. Generalized isoparametric formulation for isotropic or orthotropic FGMs material using graded elements (after Kim and Paulino [14])

Rys. 1. Uogólnione izotropowe lub ortotropowe sformułowanie dla materiałów FGM izoparametrycznego gradientowego elementu skończonego (Kim and Paulino [14])

Kim and Paulino in [14] have proposed special kind of isoparametric finite element to interpolate nonhomogeneous FGM material. They have introduced next interpolation function to approximate material properties such as Young's modulus $E = E(x)$ and Poisson ratio $v = v(x)$ as follows

$$E(x) = \sum_{i=1}^{m} N_i E_i, \quad v(x) = \sum_{i=1}^{m} N_i v_i \tag{7}$$

and it is illustrated in Fig. 1.

Kim and Paulino have assumed that the interpolation function due to material properties is the same as the shape functions for isoparametric finite element. On the other hand, the individual character of material composite constituents can be represented by a special kind of function, especially connected to the mixture rule. For example, the appropriate functions could be a power or exponential functions described below:

$$\begin{aligned} N_i(\xi,\eta) = \xi^k, \quad N_i(\xi,\eta) = \exp(k\xi), \\ N_i(\xi,\eta) = \eta^k, \quad N_i(\xi,\eta) = \exp(\kappa\eta) \end{aligned} \tag{8}$$

All things above consider, for nonhomogeneous material the stiffness matrix is given by

$$k = \int_{\Omega} (B)^T D(\xi,\eta) B \mathrm{d}\Omega \tag{9}$$

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where the constitutive matrix contains functions of local coordinate and cannot be shifted in front of the integral, in opposition to classical formulation of stiffness matrix, Eq. (6).

4. Numerical example

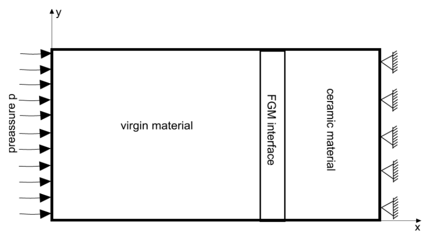

Fig. 2. Numerical model

Rys. 2. Model numeryczny

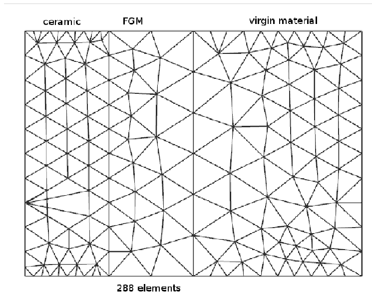

Fig. 3. Finite element mesh

Rys. 3. Siatka elementów skończonych

As an example the rectangular plane in plane stress state is considered. The numerical model is presented in Fig. 2, in detail. It is worth to noticed that the model is consisted of two materials (virgin material made of pure aluminium and Al_2O_3 ceramic phase), connected using FGM interface.

In the classical finite element formulation, the FGM interface is divided into a few parts. All of them contain constant material properties, however the material coefficients change from one to other parts (elements). Taking into account nonhomogeneous finite element formulation, where it is existed additional approximation function due to material properties, it is not necessary to divide the interface to so many times before and the FE mesh could be coarser (Fig. 3).

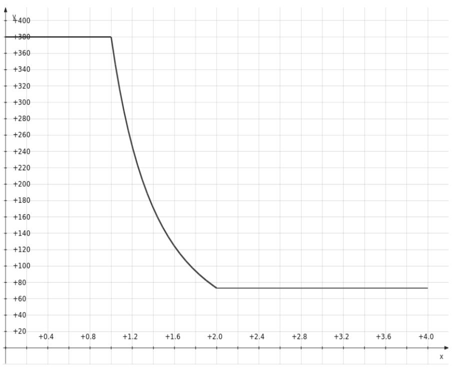

Fig. 4. Young's modulus

Rys. 5. Modułu Younga

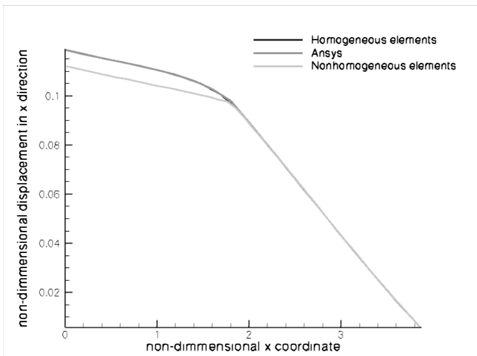

Fig. 5. Comparison of results

Rys. 4. Porównanie uzyskanych wyników

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Firstly, a numerical example was solved by commercial finite element program called Ansys. In this program, the formulation of the finite element method is classical, which means that the finite elements are homogeneous. The FGM interface was divided into five parts to interpolate the gradation of Young's modulus as it was written before. It is worth to noticed that the finite element mesh contains 434 elements.

In the next step, example was solved using own finite element code, but the finite mesh was the same as before and elements at the interface was still homogeneous. The main point of this solution is validation of numerical code.

Last solution contains nonhomogeneous finite elements at the interface and was solved by my own finite element code. However the finite element mesh was not so fine as in previous cases, because it is not necessary to divide the area, where the FGM interface is. Taking this fact into account, the amount of finite element mesh is 288. Young's modulus was approximated by power function connected to Eq. (15), however Poisson ratio υ is constant in this example. The function of Young's modulus is presented in Fig. 4.

In the Fig. 5 there is presented comparison of obtained results, which have been discussed in the previous paragraphs. Detailed analysis is shown that the reference example solved by Ansys and my own code with homogeneous elements is leading to the same result, which means that the numerical code is correct. Last line represents the solution with non-homogeneous elements. We can see a little differences between blue line and other lines in the ceramic part. This is a consequence of using nonhomogeneous finite elements and more realistic approximation of the material parameters which leads to better results.

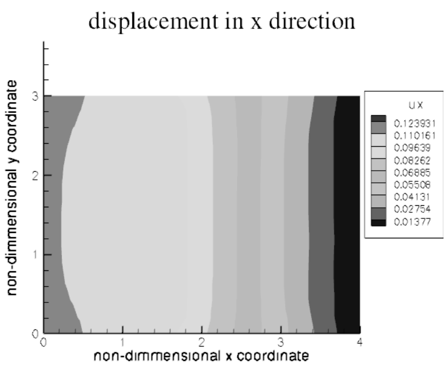

Fig. 6. Displacement map in *x* direction made of FEM with nonhomogeneous elements

Rys. 7. Mapa przemieszczeń w kierunku *x* wykonana przy pomocy kodu MES z elementami niejednorodnymi

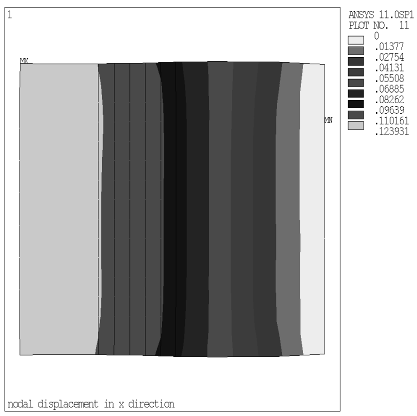

Fig. 7. Displacement map in *x* direction made of Ansys

Rys. 6. Mapa przemieszczeń w kierunku *x* wykonana w programie Ansys

In Fig. 6 and Fig. 7 there are presented maps of displacement in *x* direction. On the left hand side there is a map from the finite element code with nonhomogeneous elements and on the right side there is a map from the solution made by Ansys.

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This paper clearly shows advantageous of using the FEM with nonhomogeneous finite elements in numerical analysis of structures with FGM materials.

5. Conclusions

1. Classical finite element method with constant material properties at the each element level can not modelled FGM materials in a good way, especially, when the material homogeneity changes drastically.
2. New concept according to FGM materials of finite element formulation based on non-homogeneity FE was investigated.
3. Including the orthotropic or anisotropic materials and dependency of temperature for all material properties to non-homogeneity finite element formulation are going to improve adjustment of numerical model to the real structure.

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